BEHAVIOR OF PERIMETER BEAMS WITH INTEGRITY REINFORCING DETAILS OF LOW SEISMIC REGIONS

Jorge A. Rivera-Cruz, Sergio F. Breña, Simos Gerasimidis, and Peggi L. Clouston

Jorge A. Rivera-Cruz is a Structural Engineer at US Army Corps of Engineers, Jacksonville, FL. He received his PhD from the University of Massachusetts Amherst, Amherst, MA, in 2019; and his BS and ME degrees in Civil Engineering from the University of Puerto Rico, Mayagüez Campus, PR. His research interests include the effect of the lap splice location on behavior of beams of perimeter reinforced concrete frames after loss of an interior support and full-scale experiments.

Sergio F. Breña, FACI, is a Professor of Civil Engineering at the University of Massachusetts Amherst. He received his BS in Civil Engineering from Universidad Iberoamericana (Mexico City) in 1989, and his MS and PhD Degrees in 1990 and 2000, respectively, from the University of Texas at Austin. He is a member of ACI Committees 318-C, Safety, Serviceability, and Analysis; 318-H, Seismic Provisions; 369, Seismic Repair and Rehabilitation; and 374, Performance Based Seismic Design of Concrete Buildings.

Simos Gerasimidis is an Assistant Professor of Civil Engineering at the University of Massachusetts Amherst. He received his Diploma in Civil Engineering from the Aristotle University of Thessaloniki Greece, his Master of Engineering from MIT and his PhD from the Aristotle University of Thessaloniki Greece. His research interests include progressive collapse, infrastructure resilience, architected materials, structural instability and deteriorated bridges.

Peggi L. Clouston is a Professor of Building and Construction Technology at the University of Massachusetts Amherst. She received her BS (1989), MS (1995) and PhD (2001) at the
University of British Columbia, Vancouver, BC. She is Associate Editor of the ASCE Journal of Materials in Civil Engineering and has research interests in timber engineering, wood mechanics, and bio-based composites.

ABSTRACT

Three full-scale reinforced concrete frame sub-assemblages were tested in the laboratory to simulate the removal of an interior column. The location of bottom longitudinal reinforcement splices was varied in each specimen to evaluate ACI 318-19 detailing practices on perimeter frame beam behavior. The laboratory specimens represent two first-floor interior spans of the perimeter frame of a ten-story prototype building designed for Seismic Design Category (SDC) A. After specimens reached their peak force, a sudden decrease in force was observed due to a sudden shear failure near the exterior end of one of the beams. Failure occurred prior to development of catenary action in the beams, a phenomenon that has been observed in similar tests of specimens detailed according to higher SDCs. The limited rotation capacity of the beams in the specimens and the shear strength degradation as a result of diagonal crack widening are believed to have influenced the response of the specimens.

Keywords: structural integrity; reinforced concrete, frame building; low seismic design category; collapse loading

INTRODUCTION

Current building codes and guidelines intended to prevent progressive collapse of structures include requirements to ensure the existence of an alternate load path and adequate redundancy to mitigate the effects of progressive collapse. ASCE 7-10 (ASCE 2010) specifies two general approaches to design against the potential for progressive collapse: Direct or Indirect Design. In the Direct Design method, alternate load paths are provided in the structure
after removal of a vertical element, while the rest of the structure remains intact. In the Indirect Design method, resistance to progressive collapse is enabled implicitly by incorporating continuity between structural components and adopting ductile detailing in the design. Similar to ASCE 7-10, the *Alternate Path Analysis and Design Guidelines for Progressive Collapse Resistance* (GSA 2016 Guidelines) stipulates the alternate path method for progressive collapse resistance. The *Unified Facilities Criteria-Design of Structures to Resist Progressive Collapse* (DoD UFC 4-023-03, 2016) specifies the Direct and Indirect Design methods to determine progressive collapse resistance. The main aim of these guidelines is to reduce the potential for progressive collapse in new and renovated federal buildings.

ACI Building Code (ACI 318-19) structural integrity requirements for beams of perimeter frames are also intended to provide resistance against progressive collapse. These provisions were first introduced in the 1989 edition of ACI 318 and have remained largely unchanged since then. The provisions in ACI 318-19 §9.7.7 are intended to mitigate the potential for progressive collapse of frames by specifying longitudinal reinforcement details that foster development of alternate load paths in the event of vertical support loss. For perimeter beams, continuity of longitudinal reinforcement in ACI 318-19 (§9.7.7.5) may be achieved by providing Class B splices at or near mid-span for negative moment reinforcement and at or near the support for positive moment reinforcement. ACI 318-19 does not specify the proximity to the support region where positive moment splices may be located.

The research literature on progressive collapse is extensive, so this discussion is limited to past studies where progressive collapse behavior of reinforced concrete buildings was the main focus. Sasani and Sagiroglu (2008) conducted a field investigation on the behavior of a six-story non-ductile reinforced concrete building after explosive demolition of two columns in the first-floor corner of the building. The test demonstrated that progressive collapse was
not generated after removal of corner columns in this building because of three-dimensional frame action, even though the building contained non-ductile reinforcing details.

Yi et al. (2008) tested a four bay, three-story 0.33-scale sub-assemblage to identify the force resisting mechanism of a reinforced concrete frame after the loss of a first-floor interior column. The force-displacement response of the sub-assemblage exhibited three stages: (1) elastic response, (2) plastic response after formation of plastic hinges, and (3) catenary response with load carried through tension in the beam. Yu and Tan (2013) investigated the progressive collapse resistance of different reinforced concrete frame structures with detailing consistent with seismic and non-seismic zones. Similar to the findings from Yi et al. (2008), the researchers identified three load-carrying modes mobilized at progressively larger displacements: (1) flexural mechanism governed by the bending strength at critical sections, (2) compressive arch action developed after formation of plastic hinges, and (3) catenary action developed by beam reinforcement in tension.

Lew et al. (2014) tested two full-scale reinforced concrete beam-column sub-assemblages that simulated the removal of an interior column of a perimeter frame. Design of the sub-assemblages was based on a prototype structure with details in accordance with building designs for high or moderate seismic zones (SDC C or SDC D). The investigators divided the load transfer characteristics into three stages based on test observations: (1) arching action caused by the additional capacity provided by horizontal restraint at the beam ends; (2) plastic hinge formation governed by yielding of the reinforcing bars and concrete crushing; and (3) catenary action due to the development of tensile axial force in the longitudinal reinforcement in the beams.

Khorsandnia et al. (2017) tested two 0.4-scale reinforced concrete frames having three bays and two stories to further investigate the development of catenary action after interior
column removal. The laboratory results indicated that the beams developed minimum tensile
catenary action. Various research groups (Qian and Li, 2012; Ren et al., 2016) have studied
the increased collapse resistance the slab provides in reinforced concrete frame structures. The
experimental results showed that the presence of a reinforced concrete slab increased the
ultimate load-carrying capacity of the specimens to resist progressive collapse.

From this review of the literature, it is evident that the force carrying mechanism of
reinforced concrete frames subjected to column removal scenarios changes with the magnitude
of induced vertical displacements. Reinforcement details, presence of slab, and three-
dimensional effects are key factors that influence the development of the different behavioral
stages the frames undergo under collapse scenarios. The research study reported in this paper,
therefore, seeks to provide data to further understand the influence of reinforcement detailing
according to the current practice in ACI 318-19 on collapse resistance of perimeter frames.

RESEARCH SIGNIFICANCE

Testing results from three full-scale laboratory specimens are used to evaluate the
behavior of perimeter beams containing structural integrity details that conform to ACI 318-19
provisions for buildings in SDC A. These testing results are complementary to those by
previous research groups who have conducted tests of similar specimens but with
reinforcement detailing for higher seismic design categories. Potential development of
catenary action in beams as an alternative load-resisting mechanism to prevent progressive
collapse is evaluated on the basis of the tests presented in this paper.

LABORATORY SPECIMENS

Prototype Building Description

The same geometry of the 10-story prototype reinforced concrete building used by Lew
et al. (2014) was chosen to allow comparison with previously reported test results. The
prototype building has overall plan dimensions of 100 by 150 ft (30.5 by 45.7 m), a first story height of 15 ft (4.6 m), and a typical story height of 12 ft (3.7 m), Fig. 1. The prototype building designs were used to develop details for the laboratory specimens. The prototype buildings designed by Lew et al. (2011) corresponded to high and moderate seismic categories, whereas the prototype building in this research was analyzed and designed in accordance with *ASCE 7-10* and *ACI 318-19* for a location corresponding to a low seismic design category (SDC A). A nominal compressive strength of 4000 psi (27.6 MPa) was used for the concrete and a specified yield strength of 60 ksi (414 MPa) was assumed for reinforcement during design. Detailing the specimens in this research for SDC A, allowed comparison of observed behavioral differences of the tests with those of Lew et al. (2014).

**Laboratory Specimen Details**

The three laboratory specimens were designed and built at full-scale to capture the response of two interior spans in the first floor perimeter beam along column line 1, as shown in Fig. 1. In each specimen, the interior column was eliminated from the laboratory sub-assemblage to simulate column removal resulting from a collapse scenario, but columns with a length equal to story mid-height were built at the ends of the two beam spans.

All three specimens were identical except for the location where bottom longitudinal reinforcing bars were spliced (class B splices). The center of bottom bar splices in each specimen was located at different distances from the middle column stub. In Specimen 1 the splice center coincided with the center of the middle column, whereas the splice center was located at 2d or d from the face of the column stub for Specimens 2 and 3, respectively (d is the effective depth of the beams). In Specimens 2 and 3, bottom bars were spliced within the south span (Fig. 2). The geometry and reinforcing bar details, including bar splice locations (dashed rectangles), are shown in Fig. 2.
Table 1 lists the average compressive and tensile strengths of the concrete measured at the time of testing. Table 1 also lists values of measured mechanical properties of longitudinal reinforcing bars used in the specimens. Table 2 lists the calculated shear and moment strength of the beam at sections of maximum positive and negative moment for each specimen. These values were determined in accordance with ACI 318-19 provisions using measured material properties with a strength reduction factor equal to one.

Test Setup

A schematic drawing of the test setup and a picture of Specimen 1 positioned in the testing apparatus are shown in Fig. 3. The boundary conditions at the top and bottom of edge columns were assumed pinned, corresponding to the approximate inflection point location of columns in the prototype building between floors. Horizontal in-plane displacement at the top of the columns was restrained by providing diagonal braces fixed to the laboratory strong floor at each end. These braces, which were connected to the laboratory specimen using pin connections, were used for stability of the specimen and to provide axial restraint as would exist by the presence of a floor slab. Edge columns were supported on thick steel plates at their bases using a pin assembly anchored to the strong floor.

Four 1-in. (25.4 mm) diameter high strength threaded rods were passed through PVC pipes cast longitudinally along the edge columns. These rods were used to introduce an axial force to the columns by post-tensioning the rods between the base plate and the column top (Fig. 3b). The post-tensioning force simulated a fraction of the axial force in the columns of the prototype building, and helped secure the columns against the pinned base plates. Out-of-plane lateral support of the specimens was provided by steel elements located at 10 ft from the center of the middle column within each beam span (Fig. 3).
Load was applied using hand operated hydraulic rams throughout testing. The force was applied in increments to the column stub in the middle of the specimen that represented the location of a removed interior column. A stiff loading beam fabricated from a built-up section was used to transfer load from the rams to the specimen. Two 120 kip (534 kN) capacity hydraulic rams were placed on the top of the beam to apply force through two 1 in. (25.4 mm) diameter threaded rods connected to the ends of the beam. These rods were connected to the strong floor in the bottom by means of anchor blocks (Fig. 3).

**Instrumentation**

External instrumentation to measure the response of laboratory specimens throughout testing is shown in Fig. 4. The applied force was measured using two load cells with a capacity of 100 kip (445 kN) placed at the bottom end of each high-strength threaded rod used to apply load to the middle column stub. Also, four 50 kip (222 kN) load cells were placed in-line with threaded rods connecting the south column to the brace at that end of the specimen. These latter load cells were used to measure the horizontal force generated at the top of the south column. Seven linear displacement transducers with a 20 in. (508 mm) displacement capacity were attached at beam mid-height to measure displacements along the length of the beam with reference to the laboratory strong floor. Four inclinometers with a range of 0.524 rad (30 degrees) were attached at mid-height near each beam end to measure the rotation in regions expected to undergo plastic rotations. To measure strains of the longitudinal reinforcement along the splice regions, thirteen strain gages were attached to bottom reinforcing bars and seven were attached to top reinforcing bars along bar splices. Only selected instrumentation results are presented in this paper due to length limitations.
LABORATORY TEST RESULTS

Observed Behavior and Cracking

All specimens developed minor cracks during positioning in the test apparatus (maximum 0.005 in. [0.127 mm]). Tests were conducted by applying a vertical load on the center column. For the three specimens, the crack patterns were characterized by formation of new flexural cracks in the bottom face of the beam near the center column followed by formation of flexural cracks in the top face of the beams near the exterior columns. Diagonal cracks subsequently developed from previously formed flexural cracks in beam-end regions, primarily at the ends corresponding to exterior columns (refer to Fig. 5).

The maximum applied force in the three specimens was similar (i.e. just over 50 kip, 222 kN). At this load, evidence of initiation of concrete crushing was observed on the top surface of the beams meeting at the center column stub (compression face). Testing proceeded without being able to generate a higher applied force at increasing middle column stub displacements; widening of a critical diagonal crack near one of the beam exterior ends subsequently occurred causing the measured force to decrease suddenly because of a drastic reduction of shear strength. Final crack patterns and pictures of the critical crack at the end of testing for the three specimens are shown in Fig. 5.

Past experiments have indicated that at large vertical displacements the force carrying mechanism transitions from bending to catenary action (Lew et al. 2014; Jian and Zheng 2014) after reaching peak flexural strength. In past tests, the use of closely spaced stirrups or hoops allowed specimens to maintain their shear strength after formation and widening of diagonal cracks. The degradation of shear strength observed in specimens reported in this paper at increased vertical displacements was caused by severe widening of the critical diagonal crack.
Given the inclination of the critical diagonal crack at the exterior end of the beams, only one stirrup was effectively engaged.

**Measured Force-Displacement Response**

Figure 6 shows measured vertical force-displacement curves for the three specimens. The curves are compared with two additional curves: a solid curve, which was obtained by conducting a large-displacement nonlinear analysis using SAP2000 V. 21 (Computers and Structures, Inc., Walnut Creek, CA, 2019), and a dashed curve, which corresponds to a simplified analytical closed-form solution developed by Jian and Zheng (2014) based on the three different load transfer mechanisms that a beam-column subassemblage experiences when subjected to collapse-type loading. Details of the SAP2000 model are presented later in the *Large Displacement Plastic Analysis* Section of this paper.

The curves reveal that Specimens 1 through 3 went through the various stages associated with flexural response (cracking, yielding, peak capacity) early in the tests, indicating that load was carried through bending up to peak load. The changes in slope observed at two points prior to reaching peak load correspond to reductions in stiffness because of cracking and reinforcement yielding at increasing displacements. At peak force, the beams in all specimens reached their flexural strength and the gradual drop in force at larger displacements was caused by gradual crushing of concrete at sections of maximum moment.

For the specimens to carry additional load beyond peak as illustrated by the two calculated load-displacement curves, the load-carrying mechanism in the beam would have to transition from a flexure dominated behavior to catenary-type behavior (catenary action). Catenary action is a term that has been used in the past to identify the behavior of beams under collapse loading at large displacements because of the similarity in the way these beams resist load and the manner in which cables support transverse loading through axial forces. It is
important to note, however, that beam catenary behavior is mobilized at large displacements, so only beams with high rotation capacities are capable of withstanding large displacements without failure. Only Specimen 2 exhibited moderate load recovery after the sudden drop in force that occurred shortly after peak, pointing to initiation of catenary behavior. The other two specimens (1 and 3) were unable to develop catenary action after they experienced the sudden drop in force during testing. Horizontal force values measured at different stages during the tests (Table 3) revealed that only Specimen 2 developed any appreciable tension (34.7 kip [154.2 kN]) as a result of initiation of catenary action; the measured horizontal force values in Specimens 1 and 3 did not exceed 4.1 kip (18.2 kN).

The sudden drops in force that occurred after reaching peak in all specimens are associated with widening of a diagonal crack at the critical section (Fig. 5). The two calculated load-displacement curves presented in Fig. 6 (Jian and Zheng 2014, and SAP 2000) estimated initiation of catenary behavior at significantly different vertical displacements (10 in. [254 mm] and 21 in. [533 mm], respectively), with the curve determined using SAP 2000 better capturing the response of the specimens tested in this research. By incorporating gradual unloading of the plastic hinges used in the SAP 2000 analysis, the model was able to capture the increase in rotations that occurred because of diagonal crack widening in the specimens. The model developed by Jian and Zheng does not account directly for this additional component of rotation but rather transitions from beam behavior into catenary behavior at a specific vertical displacement value. Table 3 summarizes the measured vertical and horizontal forces and the center column stub displacement values (transducer CC-LDT) for all specimens corresponding to points labeled A through D on the curve for Specimen 2 in Fig. 6. These points are associated with specific events that were identified during testing as described below.
As summarized in Table 3, all three specimens reached approximately the same peak load (point A) at slightly different center column displacements (6.7 to 7.6 in. [170 to 193 mm]). The measured force in Specimens 1 and 2 gradually decreased at increased displacements, while Specimen 3 was able to maintain a force closer to peak prior to sudden force drop triggered by diagonal crack widening (point B). The drop in force was sudden in all specimens, but its magnitude differed. Specimens 1 and 2 had a drop of 15 and 14 kip (67 and 62 kN), respectively, while Specimen 3 had a drop of 33 kip (147 kN). Notably, the displacements in all specimens between points B and C did not increase significantly.

Differences in measured force drop values in the three specimens can be attributed to variations in diagonal crack opening in the three specimens, which in turn was influenced by relative location of transverse reinforcement along the critical diagonal crack. Reloading was subsequently attempted after the force drop associated with point C. However, only Specimens 2 and 3 were able to be reloaded to approximately 34 kip (151 kN), but the force in Specimen 1 did not increase at larger displacements. Point D corresponds to the point when testing was terminated. In Specimen 2, concerns with specimen instability generated by excessive widening of the critical diagonal crack prevented the test from continuing further. In Specimens 1 and 3, testing was stopped when it became evident that forces were not increasing at larger imposed displacements; in fact, a stirrup fractured in Specimen 3 prior to stopping the test, indicating the possibility of imminent stirrup fracture in Specimens 1 and 2 as well.

Figure 7 shows displacement plots along the span constructed using linear displacement transducers positioned along the beam (see Fig. 4) at three test stages: maximum force, widening of critical diagonal crack, and maximum displacement before failure. Cracks that widened in the beams near the exterior columns and center column after peak force localized beam rotations at these critical sections as increased vertical displacements were applied at the
middle column location. It can be noted that the post-peak deformed shapes are approximately linear between the ends of the beams, indicating that most of the post-peak (plastic) deformation was generated by section rotation occurring near beam ends. Therefore, a lumped plasticity model was deemed appropriate to estimate the response of specimens as discussed later in the paper.

**Measured Longitudinal Bar Strains in Bottom Splice Regions**

Strains were measured in selected reinforcing bars within the bottom bar splice zone for the three specimens (Fig. 8a). In each specimen within the splice zone, instrumented bars included a set of corner bars (bar 4) and a set of interior bars (bar 3). A prime symbol is used to indicate bars in the south span of the specimens that are spliced to bars that continue from the north span.

Strain gauge locations within the splice region are identified as CS, CC, or CN to indicate the location relative to the center of the splice (CS, CC, or CN for south, center, or north, respectively). The distances between these sections and the middle column centerline are indicated in Fig. 8(a). Figures 8(b) to 8(d) show the relationship between measured force and strain at the section in the splice where bars would be expected to develop their highest strain (CN section for bars entering the splice from the north [labeled B3 and B4] and CS section for bars entering the splice from the south [labeled B3’ and B4’]). In the three specimens, the spread of plasticity, approximately determined using the crack patterns observed during the tests (see Fig. 5), extended approximately 18 in. (457 mm) from the face of the middle column. This means that longitudinal bar yielding would be anticipated within a region extending into the span approximately 30 in. (762 mm) from centerline of the middle column at peak applied force, but only if bars were fully developed outside of the splice region. Bar sections CS in Specimens 2 and 3 are located outside the anticipated yielding region at 61
in. (1549 mm) and 43 in. (1092 mm), respectively, from the column centerline. These are the only two sections where bar strains were clearly below yield when the peak force was reached in the respective specimens. An examination of Figs. 8(b) through 8(d) reveals that in all other cases bars achieved yielding or were close to yielding as they exited the splice zone. Because measured strains in bottom longitudinal bars reached or exceeded yielding in the tests, it appears that locating lap splices up to a distance of $2d$ into the span from the face of the middle column achieved the goal of continuity desired of integrity reinforcement.

**EVALUATION OF SPECIMEN RESPONSE**

The response of laboratory specimens was evaluated to understand the mechanism that led to failure prior to full development of catenary action in the beams. A nonlinear plastic hinge analysis, including the effects of nonlinear geometry, was conducted to estimate the inelastic response of the beams within regions of maximum moment. The analysis included effects of plastic hinge unloading to allow capturing the post-peak response into the catenary region of behavior. Results from the nonlinear analysis, in combination with strength models that account for shear degradation at increased displacement demands, were used to better estimate the failure loads observed in the tests.

**Large Displacement Plastic Analysis**

To understand the behavior observed during the tests, an analysis was conducted that incorporated large displacements and nonlinear material behavior of the specimens using SAP 2000 (Computers and Structures). Frame elements were used to model the concrete beams and columns, and in-plane steel diagonal braces. The inelastic flexural response of the beams was captured using a lumped plasticity approach through the definition of plastic hinges in the region where inelastic action was observed during the tests. Plastic hinges were defined at
critical sections corresponding to the location of maximum negative moment near the exterior columns and maximum positive moment adjacent to the middle column stub. To define properties of flexural plastic hinges, the moment-curvature response of beam sections at the faces of exterior columns and middle column stub were calculated.

Typical moment-curvature plots for the sections of peak positive and negative moments are presented in Fig. 9. The curves are based on measured material properties (concrete and steel) and the use of an unconfined concrete model developed by Hognestad (1950). The strain corresponding to crushing of concrete was estimated as 0.0035, after which flexural strength started to degrade. A gradual post-peak degrading slope was defined to achieve analysis convergence after hinges reached their peak strength and went into the unloading regime. Values of key points identified in the moment-curvature plots of Fig. 9 (yield, peak, residual) that were used to define hinge properties for the plastic analyses are summarized in Table 4, including the assumed values used for residual flexural strength and its corresponding curvature. It should be noted that, for the purpose of conducting the plastic analysis, the moment-curvature response was simplified by eliminating the strain hardening initiation point and joining the yield point to peak point using a straight line. Stiffness reduction associated with cracking of concrete was approximated by reducing the moment of inertia of the beam to 0.3 $I_g$ and of the columns to 0.7 $I_g$, where $I_g$ is the gross moment of inertia of the cross section. The axial stiffness of the beam was decreased by reducing its gross cross-sectional area, $A_g$, to a value equal to the area of longitudinal reinforcement multiplied by the modular ratio of steel to concrete. This last adjustment was made to capture the post-peak response through cable action in the beam (tension), where stiffness is governed primarily by the steel reinforcement.

Results from the large displacement plastic analysis are shown in Fig. 6 (SAP 2000 curve) for comparison with the measured load-displacement response of the three specimens.
It can be observed that the analytical curve agrees well with the ascending portion of the diagrams up to the peak force. The gradual unloading branch in the analytical curve closely matches the measured response of Specimen 2, up to the displacement where force drop occurred suddenly because of shear failure. The sudden force drop in Specimens 1 and 3 occurred at smaller vertical displacements. The analytical curve also captures the post-peak ascending branch that takes place at larger vertical displacement after unloading. The ascending branch corresponding to catenary action was predicted to begin at about 22 in. (559 mm). Fig. 6 also shows a closed form solution developed for the collapse analysis of frames by Jian and Zheng (2014) like the ones tested in this research. Initiation of catenary behavior according to this model occurs at a much smaller displacement as can be observed in Fig. 6. The three specimens did not enter into catenary behavior at the displacement predicted by the model from Jian and Zheng.

**Shear Strength Degradation**

The observations made during the tests and the discussion in the preceding section point to the large deformation capacity needed for the specimens to develop catenary action. Specimens were designed and detailed as frames built in SDC A in accordance with ACI 318-19. The shear strength of the specimens, which was based on a building prototype designed for SDC A, was designed to avoid failure prior to reaching the beam flexural strength. As designed, all three specimens were able to reach their flexural strengths at critical sections located at the face of columns (exterior and interior). After reaching peak, the measured force decreased gradually at increased displacements following development of flexural strength. It was only after this increase in displacement that a sudden drop in measured force occurred in all specimens at a beam shear force that was much lower than calculated using ACI 318-19.
Following ACI 318-19, the concrete contribution \((V_c)\) and transverse reinforcement contribution \((V_s)\) to shear strength was calculated for each specimen (Table 2). The two procedures in ACI 318-19 to calculate \(V_c\) in nonprestressed members containing minimum transverse reinforcement are given in Eq. 1.

\[
V_c = \left[ 2\lambda \sqrt{f'_c} + \frac{N_u}{6A_g} \right] b_w d \quad (1a)
\]

\[
V_c = \left[ 8\lambda (\rho_w)^{1/3} \sqrt{f'_c} + \frac{N_u}{6A_g} \right] b_w d \quad (1b)
\]

where \(N_u\) is the factored axial load (positive for compression); \(A_g\) is the gross cross sectional area; \(f'_c\) is the concrete compressive strength (psi); \(\rho_w = A_s/b_w d\) is the longitudinal reinforcement ratio; \(\lambda\) is the lightweight concrete factor; \(b_w\) is the beam web width; and \(d\) is the effective depth to longitudinal reinforcement. Because the axial force developed in the specimens was small throughout most of the tests, \(N_u\) was neglected in these \(V_c\) calculations.

Table 2 also lists the beam shear force associated with beam flexural strength at maximum positive and negative moment sections (flexural hinging). This beam shear force at peak force is termed \(V_{plastic}\). Referring to values listed in Table 2, \(V_c\) is large enough to support the shear force at peak force corresponding to flexural plastic hinging, \(V_{plastic}\). Shear failure should, therefore, not have been anticipated. It is noted that ACI 318-19 procedures do not consider a reduction in \(V_c\) at increased beam rotations, but in order to enter into the catenary behavior range, beams must be able to achieve high rotations while maintaining shear strength.

Past researchers have recognized that the concrete contribution to shear strength \((V_c)\) degrades as higher rotation demands are placed on flexural members. Priestley et al. (1994) and Kowalsky and Priestley (2000) proposed equations to estimate \(V_c\) as a function of rotation demand in hinging regions of columns. The shear strength model they proposed estimates the total shear strength of flexural members by adding the contributions of concrete \((V_c)\), transverse
reinforcement ($V_s$), and axial force ($V_a$). Of these three terms, only $V_c$ decreases at increasing rotation (or curvature) ductility as captured by the factor $\gamma$ in Eq. (2):

$$V_c = \alpha \beta \gamma \sqrt{f'_c}(0.8A_g)$$  (2)

where $\alpha$ is a factor that accounts for the effect of shear span [$M/(Vd)$], equal to 1.0 for the beams in this study; $\beta$ accounts for the effect of longitudinal reinforcement on shear strength, also assumed equal to 1.0 here; and $\gamma$ is the factor that captures the decrease in shear strength contribution of concrete at increasing ductility demand (curvature or rotation). The reduction in $V_c$ with rotation demand is attributed to loss of aggregate interlock resulting from crack widening at large rotations. Factor $\gamma$ varies in accordance with Eq. 3a (Kowalsky and Priestley 2000) or Eq. 3b (Priestley et al. 1994).

$$\gamma = \begin{cases} 
3.5 & \text{for } \theta/\theta_y \leq 3 \\
3.5 - \frac{29}{12}(\theta/\theta_y - 3) & \text{for } 3 < \theta/\theta_y < 15 \\
0.6 & \text{for } \theta/\theta_y \geq 15 
\end{cases}$$  \hspace{1cm} (3a)

$$\gamma = \begin{cases} 
3.5 & \text{for } \theta/\theta_y \leq 3 \\
3.5 - \frac{23}{4}(\theta/\theta_y - 3) & \text{for } 3 < \theta/\theta_y \leq 7 \\
1.2 - \frac{0.6}{8}(\theta/\theta_y - 7) & \text{for } 7 < \theta/\theta_y < 15 \\
0.6 & \text{for } \theta/\theta_y \geq 15 
\end{cases}$$  \hspace{1cm} (3b)

The calculated shear force-normalized rotation relationship of the beams in Specimen 3 is shown in Fig. 10 ($V_{flex}$). Rotation demands were normalized with respect to the rotation at yield ($\theta_y$) of the beam plastic hinge sections. The two expressions for concrete contribution to shear strength ($V_c$) calculated from Eq. 2, with $\gamma$ calculated using Eq. 3a or Eq. 3b, are plotted in Fig. 10 along with $V_{flex}$. The intersections of calculated $V_c$ with the $V_{flex}$ curve correspond to estimates of beam shear and normalized rotation corresponding to diagonal crack widening in the hinge regions of the beam (Points $x$ and $y$). At these points, a drop in beam shear strength is estimated by the two models. The predicted values of shear force and normalized rotation
for shear strength loss of Specimens 1 through 3 are listed in Table 5. The model proposed by Priestley et al. (1994) resulted in lower rotation and higher shear at failure than the model by Kowalsky and Priestley (2000).

**Estimate of Rotation Demands during Tests**

Given the differences in rotation at failure obtained from the two shear strength models, it was considered appropriate to estimate the rotation demands that were placed on regions corresponding to the maximum negative moment at various stages during testing. In these regions, inelastic deformations concentrated in the specimens after yielding of the longitudinal reinforcement, which led to plastic hinge formation and eventual shear failure in the tests.

Rotation demands estimated at negative moment hinging regions were estimated at peak force (Point A) and at the force corresponding to widening of the critical diagonal crack (Point B) in Fig. 6. Rotations in hinge regions were estimated using the simple plastic model shown in Fig. 11 relying on measured values of displacement $\delta_{\text{est}}$ at the middle column stub and rotations at the exterior ends of the two beam spans ($\theta_{NN}$ and $\theta_{SS}$). Plastic hinges in the model were placed where concentration of flexural damage was predominantly observed in the tests, at 24 in. (610 mm) and 12 in. (305 mm) from the face of exterior and interior columns, respectively. In Fig. 11, the displacement $\delta_{\text{est}}$, and the rotations $\theta_{NN}$ and $\theta_{SS}$, were measured using instruments CC-LDT, NN-INC and SS-INC, respectively (see Fig. 4 for instrument location). Other displacements and rotations shown in Fig. 11 were calculated using the assumed deformed shape illustrated in the model. The rotation demands at exterior plastic hinge sections, $\theta_{\text{cal}}$, along with other displacement and rotation values used in this analysis are listed in Table 6 for Points A and B defined schematically in Fig. 6. The total hinge rotations, including the rotation estimated from beam self-weight ($\theta_{D}$), were normalized using the
rotation at yield for comparison with the normalized calculated rotations listed in Table 5 and schematically illustrated in Fig. 10.

The rotations estimated at peak force were smaller than those predicted by either model for all specimens except for the north hinge of Specimen 2, which is marginally larger than predicted by the Priestley et al. model (Point $x$). Beam rotations corresponding to Point B (diagonal crack widening) in Specimens 1 and 3 were of the same order as those predicted by the Priestley et al. model (Point $x$); beam rotations in Specimen 2 were closer to those estimated by the Kowalsky and Priestley model (Point $y$). Therefore, these two models were capable of providing reasonable estimates of the rotations anticipated to cause diagonal cracking failure (loss of $V_c$ contribution to shear strength) in the beams tested in this research. The reasonably close results obtained from comparing test-estimated rotations with those predicted using the two $V_c$ degrading models highlight the importance of accounting for degradation in $V_c$ to accurately calculate the strength and rotation capacity of beams under collapse loading scenarios.

**Influence of Diagonal Cracking Angle on Residual Shear Strength**

The loss in the $V_c$ contribution to shear strength because of widening of the critical diagonal crack resulted in a sudden drop in measured force during the tests to 28, 26, and 18 kip (125, 116, and 80 kN) in Specimens 1, 2, and 3, respectively (see Table 3). This drop in force did not result in total loss of load carrying capacity of the specimens, so it was considered important to compare the measured force after loss of $V_c$ to the estimated contribution of stirrups to shear strength ($V_s$). The calculated stirrup contribution to shear strength ($V_s$) listed in Table 2 of 52.8 kip (235 kN) corresponds to the 45 degree truss model in ACI 318-19. This value is clearly much larger than the loads at failure of the specimens (Point D, Fig. 6).
Diagonal crack angles in Specimens 1, 2 and 3 measured at the end of testing were approximately 55 degrees, 62 degrees and 63 degrees, respectively. These angles are steeper than the 45 degree truss model used in ACI 318-19, so fewer stirrups crossed the critical diagonal crack than assumed. In fact, only one stirrup typically crossed the critical diagonal crack in each specimen after loss of $V_c$. The nominal transverse reinforcement contribution to shear strength was calculated as 24 kip (107 kN), assuming one stirrup crossed the critical crack; this value was compared with the measured force after diagonal crack widening.

A dead load shear force caused by beam self-weight of 8.8 kip (39 kN) was estimated at the exterior ends; this value was subtracted from the nominal $V_s$ for one stirrup to determine the anticipated force that would correspond to beam shear failure. For the test configuration used (concentrated force at midspan), beam shear corresponds to one-half of the measured force at midspan. Forces at failure were therefore estimated as twice the difference between $V_s$ and dead-load shear (30.4 kip [135 kN]). This value is close to the measured force values at Point D in the three specimens (see Table 3). This result suggests that the inability of specimens to develop any significant catenary action was caused by a complete loss of shear strength ($V_c$ and $V_s$).

**Influence of Transverse Reinforcement Spacing on Development of Catenary Behavior**

The specimens tested in this research study did not behave as reported by those tested by Lew et al. (2014), where catenary action developed in the beams after formation of diagonal cracks near beam ends. Lew et al.’s specimens were designed following details of special moment resisting frames in high seismic zones, which specify closely spaced hoops at a maximum spacing of $d/4$ within hinging regions. The specimens tested by Lew et al. failed after fracture of bottom reinforcing bars near a flexural crack that formed adjacent to the center column. The difference in behavior with the specimens reported in this paper highlight the
importance of using closer transverse reinforcement spacing than currently permitted for beams in SDC A to allow development of catenary action in beams of frames subjected to collapse-type scenarios.

CONCLUSIONS

Based on the results of this research, the following conclusions are made:

1. To provide continuity of bottom longitudinal reinforcement in perimeter beams, splices can be located within the beam-column joint region or within a region of $2d$ adjacent to the face of the column. Splice location did not affect the behavior of the specimens tested in this research.

2. The sudden reduction in shear strength that specimens experienced during the tests was attributed to the degradation in the concrete contribution to shear strength ($V_c$) at large rotations. Expected rotation demands should be considered when estimating shear strength at high rotations to ensure that sufficient transverse reinforcement is provided and to properly detail beams in perimeter frames for collapse resistance.

3. Although catenary action has been identified as a load carrying mechanism developed at large displacements of beams under collapse-type scenarios, the specimens tested in this research were not able to go into this range of behavior. Because of the limited rotation capacity of the beams tested in this research, the observed behavior contrasts with findings from previous research studies that demonstrated the ability of beams containing seismic details to resist loads through catenary action at large vertical displacements.

4. Detailing of transverse reinforcement in hinging regions at the end of beams is critical to avoid sudden loss in shear strength and to be able to develop higher forces after diagonal cracking. This research and results from prior tests of perimeter beams containing seismic details suggest that closer spacing of transverse reinforcement is needed to avoid shear
strength loss and to enable development of higher forces through catenary action at increased beam rotations.

5. Based on the results and discussions presented in this paper, it is recommended that the maximum spacing of transverse reinforcement at both ends of perimeter beams be limited to \( d/4 \) over a length of twice the beam depth measured from the face of the column toward midspan. It is also recommended to avoid top bar cutoffs in tension zones at the ends of perimeter beams by taking into account the shift in location of the point of inflection that occurs after an interior support is lost.

ACKNOWLEDGMENTS

The first author expresses his gratitude to The Concrete Reinforced Steel Institute (CRSI) and the Northeast Alliance for Graduate Education and the Professoriate (NEAGEP) for supporting this research and his PhD studies.

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Figure 10 – Comparison of plastic shear with calculated shear strength at exterior ends of north and south spans.

Figure 11 – Simplified plastic model to calculate rotation at inelastic hinge locations
### Table 1 – Measured material properties of concrete and longitudinal reinforcing bars

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Avg. ( f'c ), psi (MPa)</th>
<th>Avg. ( f_i ), psi (MPa)</th>
<th>( f_y ), ksi (MPa)</th>
<th>( f_u ), ksi (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. 6 No. 7 No. 6 No. 7</td>
<td>No. 6 No. 7 No. 6 No. 7</td>
<td>No. 6 No. 7 No. 6 No. 7</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5230 (36.1) 360 (2.5)</td>
<td>67 (462) 69 (476)</td>
<td>104 (717) 110 (758)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4140 (28.5) 360 (2.5)</td>
<td>67 (462) 69 (476)</td>
<td>104 (717) 110 (758)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5080 (35.0) 460 (3.2)</td>
<td>77 (53.9) 69 (476)</td>
<td>104 (717) 110 (758)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2 – Calculated moment and shear strength of beam sections

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( M_n^{(\pm)} ), kip-ft (kN-m)</th>
<th>( M_n^{(\mp)} ), kip-ft (kN-m)</th>
<th>( V_{plastic} ), kip (kN)</th>
<th>( V_c ), kip (kN) [Eq. 1a]</th>
<th>( V_e ), kip (kN) [Eq. 1b]</th>
<th>( V_s ), kip (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End (A-A)*</td>
<td>Midspan (B-B)*</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>232.5 (315.3) 118.9 (161.2)</td>
<td>167.8 (227.5) 26.7 (118.8)</td>
<td>61.2 (272) 43.6 (194)</td>
<td>52.8 (235)</td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>229.7 (311.5) 118.2 (160.3)</td>
<td>166.3 (225.5) 26.4 (117.4)</td>
<td>54.4 (242) 38.8 (173)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>232.2 (314.9) 118.9 (161.2)</td>
<td>167.6 (227.3) 26.7 (118.8)</td>
<td>60.3 (268) 43.0 (191)</td>
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</tr>
</tbody>
</table>

*see Figure 2

### Table 3 – Measured force-displacement* values at various test stages

<table>
<thead>
<tr>
<th>Point**</th>
<th>Specimen 1</th>
<th>Specimen 2</th>
<th>Specimen 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Specimen 1</td>
<td>Specimen 2</td>
<td>Specimen 3</td>
</tr>
<tr>
<td></td>
<td>( P ) (kip)</td>
<td>( \delta_{est} ) (in.)</td>
<td>( H ) (kip)</td>
</tr>
<tr>
<td>A</td>
<td>52.4</td>
<td>6.7</td>
<td>0.2</td>
</tr>
<tr>
<td>B</td>
<td>42.1</td>
<td>13.9</td>
<td>3.8</td>
</tr>
<tr>
<td>C</td>
<td>28</td>
<td>14.1</td>
<td>0.5</td>
</tr>
<tr>
<td>D</td>
<td>28</td>
<td>16.3</td>
<td>1.4</td>
</tr>
</tbody>
</table>

*\( P \) is the measured vertical force; \( H \) is the measured horizontal force; \( \delta_{est} \) is the measured vertical displacement

**Point A – peak vertical force; Point B – widening of shear crack; Point C – force after shear crack widening; Point D – end of test (Note: 1 kip = 4.448 kN; 1 in. = 25.4 mm)
### Table 4 – Plastic hinge parameters used in large-displacement nonlinear analysis of specimens

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Exterior negative moment, $M^{(-)}$</th>
<th>Interior positive moment, $M^{(+)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location relative to face of column, in (mm)</td>
<td>24 (610)</td>
<td>12 (305)</td>
</tr>
<tr>
<td>Length of plastic hinge, in. (mm)</td>
<td>20 (608)</td>
<td>20 (608)</td>
</tr>
<tr>
<td>Yield moment, $M_y$, kip-ft (kN-m)</td>
<td>225.0 (305.1)</td>
<td>162.1 (219.8)</td>
</tr>
<tr>
<td>Peak moment, $M_{pk}$</td>
<td>1.16 $M_y$</td>
<td>1.15 $M_y$</td>
</tr>
<tr>
<td>Residual Moment, $M_{res}$</td>
<td>0.3 $M_y$</td>
<td>0.3 $M_y$</td>
</tr>
<tr>
<td>Curvature at yield, $\varphi_y$, 1/in. (1/m)</td>
<td>0.00022 (0.0087)</td>
<td>0.00022 (0.0087)</td>
</tr>
<tr>
<td>Curvature at peak, $\varphi_{pk}$</td>
<td>7.2 $\varphi_y$</td>
<td>8.5 $\varphi_y$</td>
</tr>
<tr>
<td>Curvature at residual strength, $\varphi_{res}$</td>
<td>50 $\varphi_y$</td>
<td>50 $\varphi_y$</td>
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### Table 5 – Estimated beam shear force and normalized rotations at failure (Fig. 10)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Point $x$ [Eq. 2, 3b]</th>
<th>Point $y$ [Eq. 2, 3a]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam shear, $V_b$ (kip)</td>
<td>Force*, $P_{calc}$ (kip)</td>
</tr>
<tr>
<td>1</td>
<td>27.0</td>
<td>44.5</td>
</tr>
<tr>
<td>2</td>
<td>29.6</td>
<td>49.7</td>
</tr>
<tr>
<td>3</td>
<td>28.0</td>
<td>46.5</td>
</tr>
</tbody>
</table>

*Note: $P_{calc} = 2(V_b - V_D)$; $V_D = 4.75$ kip; 1 kip = 4.448 kN
Table 6 – Calculated rotations of beam plastic hinge sections

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$\delta_{test}$, in.</th>
<th>Beam end</th>
<th>$\theta_{SS}$ or $\theta_{NN}$, rad</th>
<th>$\delta_1$, in.</th>
<th>$\delta_2$, in.</th>
<th>$\theta_{cal}$, rad</th>
<th>$\theta_{cal} + \theta_D$</th>
<th>$	heta_y$</th>
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<tr>
<td>Peak Load (Point A, Fig. 6)</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.7</td>
<td>South</td>
<td>0.015</td>
<td>0.36</td>
<td>6.3</td>
<td>0.033</td>
<td>0.018</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>North</td>
<td>0.015</td>
<td>0.36</td>
<td>6.3</td>
<td>0.033</td>
<td>0.018</td>
<td>4.3</td>
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<tr>
<td>2</td>
<td>7.6</td>
<td>South</td>
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<td>0.26</td>
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<td>0.038</td>
<td>0.027</td>
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<td></td>
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<td>0.039</td>
<td>0.034</td>
<td>7.9</td>
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<tr>
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<td>7.4</td>
<td>South</td>
<td>0.014</td>
<td>0.34</td>
<td>7.1</td>
<td>0.037</td>
<td>0.023</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>North</td>
<td>0.015</td>
<td>0.36</td>
<td>7.0</td>
<td>0.037</td>
<td>0.022</td>
<td>5.1</td>
</tr>
<tr>
<td>Diagonal Crack Widening (Point B, Fig. 6)</td>
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<td></td>
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<tr>
<td>1</td>
<td>13.9</td>
<td>South</td>
<td>_§</td>
<td>_§</td>
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<td>_§</td>
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<tr>
<td></td>
<td></td>
<td>North</td>
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<td>0.018</td>
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<td>0.062</td>
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<td>0.081</td>
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<td>0.052</td>
<td>0.033</td>
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<tr>
<td></td>
<td></td>
<td>North</td>
<td>0.019</td>
<td>0.46</td>
<td>10.0</td>
<td>0.052</td>
<td>0.033</td>
<td>7.8</td>
</tr>
</tbody>
</table>

*§Instrument malfunction; $*\theta_D = 0.0008$ rad, $\theta_y = 0.0044$ rad
(a) Typical floor plan  
(b) Elevation along column line 1

**Fig. 1** – Prototype building dimensions (Lew et al. 2014); specimen geometry shown shaded in part (b). (Note: 1 ft = 12 in. = 304.8 mm)

**Fig. 2** – Specimen geometry and reinforcing bar details. Dashed box indicates location of bottom bar splice. (Note: 1 ft = 12 in. = 305 mm)
(a) Schematic of laboratory setup

(b) View of laboratory setup (Specimen 1 shown)

**Fig. 3** – Test Setup details. (Note: 1 ft = 12 in. = 305 mm)

**Fig. 4** – External instrumentation details: LC=load cell; INC=inclinometer; LDT=linear displacement transducer. (Note: 1 ft = 12 in. = 305 mm)
Fig. 5 – Observed crack patterns and critical crack at end of the test. (Note: 1 in. = 25.4 mm)

Fig. 6 – Measured and calculated vertical force-displacement response
Fig. 7 – Deformed shape of the beams at peak, post-peak after widening of critical diagonal crack, and end of test

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**Fig. 9** – Calculated moment-curvature response of beam sections within plastic hinge regions

**Fig. 10** – Comparison of plastic shear with calculated shear strength at exterior ends of north and south spans. (Note: 1 in. = 25.4 mm)

**Fig. 11** – Simplified model to calculate rotation at inelastic hinge locations
(Note: 1 in. = 25.4 mm)