1 Introduction

From aircraft fuselages to roofs of large buildings and from boat hulls to tall wind turbine towers, thin shell structures are used extensively due to their structural efficiency and appeal to aesthetics. Owing to the ease of construction, thin cylindrical shells occupied a prominent place among thin shell structures. However, the high sensitivity of thin cylindrical shells to imperfections diminishes their structural efficiency and induces an element of uncertainty. The presence of imperfections in thin cylindrical shells reduces their load carrying capacity significantly and extensive studies have been followed to understand this peculiar nature [1–10]. All thin cylindrical shells always contain imperfections in one form or another and thus their actual capacities are significantly lower than their theoretical capacity, i.e., the load they would have been able to uphold without imperfections.

In the light of high sensitivity of thin cylindrical shells to imperfections, almost all design codes [11,12] use the knockdown factor approach. In this approach, the theoretical capacity of thin cylindrical shells is reduced by a factor, known as the knockdown factor and the resulting value is assigned as the basis for calculating the design capacity. This way, the effect of imperfections is incorporated into the design. In Fig. 1(a), the experimental values of axial compressive capacity and bending capacity of thin cylindrical shells are plotted against radius to thickness ratio (R/t) along with the lower bound curves. These lower bound curves are the empirical knockdown factor and they are used to calculate the design capacity. In Fig. 1(b), the empirical knockdown factor for axial compression and bending is plotted separately for more clarity. The capacity of thin cylinders is reduced significantly, around 41% and 32% for axial compression and bending load scenarios, respectively, for R/t = 100. European codes [12] also recommend similar reductions based on construction quality. These reductions are conservative but necessary, as many uncertainties are associated with the shape and the size of imperfections.

With the developments in manufacturing processes and the sophistication in construction, the empirical formulas for the knockdown factor could potentially be reconsidered [14] and become less conservative. With these improvements, the quality of manufactured cylinders is improved, but the sensitivity of cylinders to imperfections is still there. In other words, advanced manufacturing techniques are reducing the presence and amplitude of imperfections, but they are not eliminating the sensitivity of thin cylinders to imperfections. An alternative way to increase the capacity of thin cylinders and to reduce their sensitivity to imperfections is the use of stiffeners along the axial and circumferential directions. However, these stiffeners increase the required volume of material and the cost of construction [14]. In all the attempts to improve the design of thin cylinders, the fundamental problem which is the sensitivity to imperfections is always present and dominating.

Recently, an alternative approach has emerged to reduce the imperfection sensitivity and to increase the load carrying capacity of thin cylindrical shells [14–17]. Wavy cross-sectional shapes are explored instead of circular cross sections. The wavy cross-section shape reduces the effective slenderness (R/t) of cylindrical shells as the local radius of curvature is reduced and consequently, the imperfection sensitivity of thin cylindrical shells is also reduced. Furthermore, the stiffness developed by the waviness and the periodic change in the direction of curvature also play an important role to make wavy cylinders insensitive to imperfections. This approach changes the characteristic of thin cylindrical shells fundamentally, and consequently, the response of thin cylinders to imperfections is altered.
Ning and Pellegrino [14,15] have carried out a computational and experimental study to investigate the imperfection sensitivity of wavy shape cross-sectional cylindrical shells and they have presented highly promising results. In their computational work, thin cylinders are subjected to axial compression; the material is linear elastic and the shape of imperfection is the first eigenmode. In their experimental work, they have validated this new approach by measuring the geometric imperfection by a photogrammetry technique and then incorporating the imperfection in the computational models. These experiments confirmed that the wavy shells are not sensitive to imperfections and have put thin cylindrical shells in a new light which has opened up new avenues of exploration.

This study presents new insights into the response of wavy cylinders under uniform axial compression and bending. The first part of this paper focuses on thin wavy cylindrical shells under axial compression, similar to the groundbreaking papers by Ning and Pellegrino. The presented work in this paper is differentiated from the previous work by the shape of geometric imperfections. The shape of imperfections chosen for this purpose is a local dimple-like imperfection, which is more realistic than the first eigenmode. The second part of this paper which can be considered as the main contribution of this study extends the investigation of wavy cylindrical shells to cylinders under bending, as bending is the primary load in several important applications of thin cylinders, e.g., tall wind turbine towers and gas pipelines. In these applications, for the typical R/t ratio used, the cylinders fail inelastically [18]. The results of Ning and Pellegrino [14,15] cannot be extrapolated directly to the cylinders under bending and thus a thorough study is needed to evaluate the effectiveness of wavy cylinders under bending considering the inelastic response of the material. So that wavy cylinders can be used confidently for several important applications where the primary load is bending and the material behavior is not linear elastic.

The instability of thin cylindrical shells under bending is a complex phenomenon due to the interaction of two different modes of failure, namely, ovalization [19] and bifurcation [20]. This interaction makes the bending of thin cylindrical shells a challenging and interesting mechanics problem [18]. Material plasticity and the presence of imperfections add more to this complexity. We will not go in that direction in detail; interested readers are directed to Refs. [13,18–22]. We focus on the effect of imperfections on thin wavy cylinders under bending and the main hypothesis to be answered; are they as effective as in the case of elastic axial compression [14,15] or not? The industrial potential of such a solution could be significant based on the applications of thin cylindrical shells under bending.

We choose a typical thin cylindrical shell which represents a section of a tall wind turbine tower studied by the authors in a previous paper [18], and we modify it into a wavy shell. After that, we introduce geometric imperfections in the cylinders and observe their impact. Two types of geometric imperfections are considered: biased sinusoidal shaped [21] and dimple-like imperfections. For the material model, a version of the Ramberg–Osgood strain hardening model is utilized.

We are reporting very encouraging results; thin wavy cylinders under bending and the inelastic range of material are imperfection insensitive especially when the amplitude of imperfection is small (w<sub>c</sub><0.3). In addition, the effect of wave parameters, e.g., the amplitude and the number of waves, on the bending behavior is explored. We found that wave parameters play a decisive role in making thin wavy cylinders imperfection insensitive.

This paper is organized in the following way. In Sec. 2, the geometry of the thin wavy cylinders used in this study is described. In Sec. 3, we examine the mechanism behind imperfection insensitivity of wavy cylinders under axial compression assuming linear elastic material behavior. The effects of imperfections on thin wavy cylinder under bending are discussed in Sec. 4 along with the influence of the wave parameters. In the same section, the comparison is done between thin wavy cylinders and stiffened cylinders under bending to evaluate their effectiveness. Finally, the paper is concluded in Sec. 5 with the main findings.

2 Geometry of Thin Wavy Cylinders and Finite Element Modeling

There are many ways to create a wavy cross section of a cylinder. We choose a simple way to create wavy cross sections in which a sinusoidal wave is superimposed on the circular cross section with a specified wavelength and amplitude. Figure 2 shows the cross sections of circular and wavy cylinders along with their analytical equations. The wavy cross section is created by superimposing a sine wave, with amplitude A<sub>r</sub> and number of waves N, on the circumference of the circular (called base circle in Ref. [14]) cross section, whose radius is R.

This wavy cross section is fully characterized by three parameters: the radius of the base circle R, the wave amplitude A<sub>r</sub>, and the number of waves N. The analytical equation of the wavy cross section is [14]

\[ r(θ) = R + A_r \sin(Nθ) \]  

where θ is the polar coordinate, R is the radius of the base circle, A<sub>r</sub> is the amplitude of the wave, and N is the number of waves along the circumference.

All the analyses are performed in ABAQUS [23] using the arc-length-based Riks method [24]. For meshing, four-node reduced integration shell (S4R) elements are created by user-written codes with an element size 121.20 mm, about 0.61√R, in both
axial and circumferential directions. Four integration points were utilized along with the thickness of each element. Figure 3 demonstrates a representative finite element model. Two nodes are defined at the center of the top and bottom cross sections of the cylinder; we call them center nodes. Rigid links are created to connect the nodes at the end of the cylinder to the respective center nodes to constrain the displacements $U_1$, $U_2$, and $U_3$ and rotations $UR_1$, $UR_2$, and $UR_3$ of the nodes at the end from moving and rotating with respect to the center nodes. Using these constraints, one end of the cylinder is clamped by fixing the central node at $z = 0$. At the other end ($z = L$), a clamped boundary condition is enforced, but the end of the cylinder is loaded by applying an axial displacement, $U_3 = -\Delta$, for the cylinder under axial compression and by applying a rotational displacement, $UR_1 = 0$, for the cylinder under bending.

The material properties and the dimensions of the cylinders are described in later sections separately for axial compression (linear elasticity) and bending (plasticity).

### 3 Wavy Cylindrical Shells Under Axial Compression

In this section, the effect of a dimple-like imperfection is studied to evaluate the effectiveness of wavy cylinders under axial compression. Dimple-like imperfection is a more realistic imperfection because it can be induced easily when thin cylinders are hit by some sharp solid objects—usually during transportation and construction [18,25–27]. Although previous pioneering studies [14,15] have observed the response of wavy cylinders to imperfections under axial compression, the imperfections had not been realistic from an application aspect (they are in the shape of the first eigenmode). We also examine the mechanism behind the imperfection insensitivity of wavy cylinders; this is the main objective of this section and thus we assume linear elastic material behavior for simplicity purposes. The dimensions and the material properties of the cylinder, which are used in this section, are given in Table 1. These are based on the work of Yadav and Gerasimidis [18] focused on bending of thin cylinders except for the length $L$. The value of length $L$ is taken as $4R$ based on Gerasimidis et al.’s [25] work on thin cylinders under axial compression. These dimensions are common in the industry of steel wind turbine towers.

The wave amplitude $A_r$ and the number of waves $N$ of Eq. (1) will be used as variables to explore their effect on imperfection sensitivity.

#### 3.1 Sensitivity to Dimple-Like Imperfections.

Local dimple-like imperfections are commonly induced in thin cylinders when they are hit by some sharp solid. We apply a single dimple that is placed at the middle of the cylinder. To model dimple-like imperfections, we use what was proposed by Yadav and Gerasimidis [18] and Gerasimidis et al. [25]. The mathematical description of the dimple-like imperfection is

$$w = -w_0 e^{-\frac{(z/L)^2}{2}} e^{-\frac{(r/R)^2}{2}}$$

where $w$ represents the deviation of the shell surface from the original position in the radial direction and $w_0$ is the amplitude of the imperfection; $x$ and $\theta$ are the axial and circumferential coordinates with the origin placed in the middle of the cylinder. Figure 4 shows this dimple-like imperfection, which is placed at the middle of the cylinder, along with the coordinates $x$ and $\theta$, $L_1$ and $\theta_1$ are the parameters which decide the length (in the axial direction) and width (in the circumferential direction) of the dimple. The values for $L_1$ and $\theta_1$ are chosen such that the length ($2L_1$) and the width ($2\theta_1$) of the dimple are equal to the first eigenmode wavelength of the circular cylinder under axial compression, i.e., $3.44\sqrt{R}$ for $\nu = 0.3$ [28]. This is done in anticipation that this length and width of the dimple-like imperfection reduce the capacity the most [18]. We use this dimple-like imperfection in both the circular and wavy cylinders and compute the reduction in their load carrying capacities. For wavy cylinders, the number of waves $N$ is 15 and the wave amplitude $A_r$ is taken as $R/70$ (1.7t), $2R/70$ (3.4t), and $3R/70$ (5.1t). We vary the amplitude of imperfections $w_0$ from 0 to $t$ because for most practical purposes, the amplitude of imperfections is not more than the thickness of the cylinder.

Figure 5 shows the knockdown factor for the circular and wavy cylinders against imperfection amplitude. The knockdown
Factors are always higher for wavy cylinders (less reduction) as compared to the circular cylinder and the effectiveness of the wavy cylinders in terms of insensitivity to imperfection increases with the increase of the wave amplitude $A_w$. This is evidence that the wavy cylinders are not only insensitive to eigenmode imperfections as reported by Ning and Pellegrino [15,16] but also to dimple-like imperfections.

Many significant observations can be made from Fig. 5. First, the wave amplitude drastically affects the behavior of the cylinders as the knockdown factor in case of $A_w = 2R/70 (3.4t)$ and $A_w = 3R/70 (5.1t)$ is almost 1. For $A_w = R/70 (1.7t)$, the knockdown factor is significantly less than 1 but still more than that of the circular cylinder. Second, knockdown factors stabilize for all the cases at higher imperfection amplitudes and further increase in imperfection amplitude does not further reduce load carrying capacities. Stabilization in knockdown factor is also reported in many past studies [18,26,27,29,30]. Third, for the wavy cylinders, the plateaus occur at smaller imperfection amplitudes.

The imperfection insensitivity of wavy cylinders is attributed to many factors. The first is the reduction in the local radius of curvature. Due to the waviness, curvatures along the circumference are reduced significantly and thus the effective $R/t$ drops. We know the imperfection sensitivity of shells reduces with the reduction of $R/t$ ratio (Fig. 1). To understand how $R/t$ ratio affects the imperfection sensitivity, we plot the knockdown factor for circular cylindrical shells in Fig. 6(a) with varying $R/t$ ratio from 120 to 10. The thickness of the cylinder is constant and equal to 2/120 m for all these cylinders; we are varying only radius $R$ to vary the $R/t$ ratio. The local dimple-like imperfections are not scaled; this means that the length and width of the dimple are same for all the cases. It can be observed that the knockdown factor is increasing with the reduction of $R/t$ ratios, keeping imperfection the same (unscaled). Keeping imperfection unscaled is important to see the impact of $R/t$ on knockdown factors, otherwise if we scale the imperfection with $R/t$, knockdown factors will remain unaffected [26]. In Fig. 6(b), we plot wavy cylinders with $A_w = R/70 (1.7t)$, $2R/70 (3.4t)$, and $3R/70 (5.1t)$ and the circular cylinder with $R/t = 120, 48$, and 10. For wavy cylinders, the radius of the base circle is 2 m, $N = 15$, and $R/t$ ratio is 120. For wavy cylinders with $A_w = 2R/70 (3.4t)$ and $A_w = 3R/70 (5.1t)$, the knockdown factors are even higher than the knockdown factor for circular cylinder with $R/t = 10$. In addition, the knockdown factor variation with the imperfection amplitude of wavy cylinders is different than that of circular cylinders. For wavy cylinders, the knockdown factor does not vary with imperfection amplitude substantially, whereas, for circular cylinders, the knockdown factor decreases with the imperfection amplitude. The reduction in the $R/t$ ratio reduces the knockdown factor but does not alter the high sensitivity of circular cylinders to imperfections. Therefore, the reduction of the local radius of curvature is not the only reason for the insensitivity of wavy cylinders to imperfections, as given by Ning and Pellegrino [15], although this is the primary reason.

The second attribute to the imperfection insensitivity of wavy cylinders is the stiffness developed by the waviness along the axial direction. An important feature of Fig. 6 is the drastic difference in knockdown factors of wavy cylinders by changing the wave amplitude $A_w$. This is happening because by changing the wave amplitude, the stiffness of the cylinder is changed notably. To understand this more clearly, we show the first eigenmode of circular and wavy cylinders in Fig. 7. It can be seen that for smaller wave amplitude, $A_w = R/70 (1.7t)$, the eigenmode of the wavy cylinder is similar to the circular cylinder. This is happening because for small wave amplitude, the stiffening effect of the cylinder is altered only slightly and thus, the increase in imperfection insensitivity, in this case, is not as pronounced as with the other wavy cylinders. For wavy cylinders with $A_w = 2R/70 (3.4t)$ and $3R/70 (5.1t)$, the first eigenmode is quite different than the circular cylinder and resembles more to the circular cylinder with stiffeners along the axial direction. Consequently, knockdown factors are increased considerably.

### 3.2 Sensitivity to Axisymmetric Imperfections

We create an axisymmetric imperfection by eliminating the circumferential
variation, $e^{-0.01b^2}$, from Eq. (2):

$$w = -w_o e^{-0.01b^2}$$ (3)

In this equation, the imperfection depends only on the axial coordinate $x$, and its amplitude is the same along the circumference. All the other parameters of Eq. (3) have the same meaning as in Eq. (2). The direction of the imperfection is always inward (toward the center of the cross section) while the direction of curvature of the wavy cross section is changing direction periodically from inward to outward. We know from previous studies on axisymmetric imperfections, the imperfections are surprisingly not reducing but increasing the load carrying capacity of the wavy cylinder within the range of imperfection amplitudes considered. This surprising result is an indication that longitudinal waves (here coming from imperfection) could potentially be beneficial for the capacity, if combined with circumferential waves.

To further investigate this issue, we imposed an antisymmetric imperfection in the wavy cylinder, which is aligned with the curvature direction of the wavy cross section; i.e., if the direction of the curvature is inward then the imperfection is outward, and if the direction of the curvature is outward, then the imperfection is inward (Fig. 10). To create such an imperfection pattern, we modified Eq. (2) by adding a $\sin(N\theta)$ term in the equation. The value of $N$ is kept 15 as for the wavy cross section. We call this imperfection as harmonized. The mathematical equation of this imperfection is

$$w = -w_o e^{-0.01b^2} \sin(N\theta)$$ (4)

Figure 9 shows the harmonized imperfect wavy cylinder along with the cross sections at the middle of the perfect and imperfect wavy cylinders. Again, we have scaled up the imperfection

Fig. 8 Disharmonized axisymmetric imperfections. When the curvature is inward, the imperfection is also considered inward which is the most deleterious case. However, when the curvature of the wavy shell is outward, the imperfection is outward and therefore its effect is not as deleterious as in the first case.

Fig. 9 Knockdown factor for wavy cylinders with harmonized and disharmonized axisymmetric imperfections. For disharmonized axisymmetric imperfections, the imperfections are surprisingly not reducing but increasing the load carrying capacity, indicating that a two-directional wave could be beneficial for the cylinder.
amplitude in this figure for visual clarity. The knockdown factor is also shown in Fig. 9. The knockdown factor, for harmonized imperfection, is reducing with increasing imperfection amplitude, which is a contrast to the response of the wavy cylinder to the disharmonized imperfection. This is happening because there is no resistance to radial displacement in this case and thus the behavior is as expected, i.e., the knockdown factor reduces with the increase of imperfection amplitude $w_i$. However, the wavy cylinder is still rather insensitive to imperfection. Harmonized imperfection is quite artificial as the direction of imperfection is inward or outward. Thus, we have another benefit of using wavy cylinders, which further makes them insensitive to imperfection.

In summary, the mechanism behind the imperfection insensitivity of wavy cylinders can be described by the following observations:

1. The first observation, the most significant one, is that the reduction in effective radius of curvature, which consequently reduces the effective $R/t$ ratio, renders wavy cylinders less sensitive to imperfections.
2. Second is the stiffness developed by the waviness of wavy cylinders. Waviness also works as a stiffener, which can be seen by the change of the first eigenmode with the increase in the wave amplitude (Fig. 7).
3. The third is the periodic change in the direction of the curvature of wavy cylinders: the direction of curvature changing from inward to upward. Often, the direction of a real imperfection is inward or outward but not in both directions. Thus, there is disharmony between the curvature direction and the direction of the imperfection. This provides a kind benefit, which makes wavy cylinders further insensitive to the imperfections.

These three factors work simultaneously and make wavy cylinders insensitive to imperfections.

### 3.3 Verification of Computational Findings

To verify our numerical analysis and the key finding, i.e., the insensitivity of wavy cylinders to imperfections, we compared our numerical results with experimental data of Ning and Pellegrino [15] as these are the only experiments on the imperfection sensitivity of wavy cylinders. The dimensions and the material properties used in the experiments are different from the dimensions and the material properties used in this study. For this reason, we modified the dimensions and the material properties of our analyses and compared the results of our computational analysis with the experimental data. The length $L$, radius $R$, and the wave amplitude $A_i$ of the three experimental samples are 70 mm, 35 mm, and 1.5 mm, respectively. The nominal shell thickness $t$ of the samples is around 180 $\mu$m, and the measured thickness of the three samples is $166 \pm 16$ $\mu$m, $166 \pm 22$ $\mu$m, and $165 \pm 19$ $\mu$m. The experimental samples are composites; interested readers are referred to Ref. [15] for the ABD matrix and other details. The measured amplitudes of the mid-surface imperfections of the three samples are 2.08$\text{t}$, 2.53$\text{t}$, and 2.98$\text{t}$. The computational results are in accordance to the experimental findings of Ref. [15] and both reveal an insensitivity of the wavy cylinders to imperfection amplitude.

### 4 Wavy Cylinders Under Bending

To study the imperfection sensitivity of wavy cylinders under bending, we choose the dimensions and material properties as taken by Yadav and Gerasimidis [18] and given in Table 3. One end of the cylinder is fixed and rotation is applied at the other end (see Sec. 2 for details). For the stress–strain relationship, a version of the Ramberg–Osgood stress–strain relationship is used [18]

$$\varepsilon = \frac{\sigma}{E} \left[ 1 + \frac{3}{7} \left( \frac{\sigma}{\sigma_y} \right)^n \right]^{-1}$$  \hspace{1cm} (5)

Two rigid body constraints are imposed at the end cross sections, which make sure that the end cross sections do not change their

| Table 2 Computation results from current analysis compared to the experimental failure loads from Ref. [15] |
|---|---|
| **Sample** | **Computational results of current study (kN)** | **Experimental failure loads from Ref. [15] (kN)** |
| Sample 1 | 10.9411 | 11.48 ± 0.03 |
| Sample 2 | 10.9403 | 11.68 ± 0.03 |
| Sample 3 | 10.9394 | 11.30 ± 0.03 |

Note: Sample 1 has an imperfection with an amplitude of 2.08$\text{t}$, sample 2 with an amplitude of 2.53$\text{t}$, and sample 3 with 2.98$\text{t}$. The computational results are in accordance to the experimental results of Ref. [15] and both reveal an insensitivity of the wavy cylinders to imperfection amplitude.
shape, i.e., ovalization is prevented during the analysis. We chose these dimensions and material properties because they are typical sections of tall and super-tall wind turbine towers and for these cylinders, the stresses cross the yield stress limit before buckling [18].

4.1 Axisymmetric Disharmonized Imperfections. To study the response of thin wavy cylindrical shells under bending to imperfections, a sinusoidal axisymmetric disharmonized geometric imperfection is used first. The amplitude of this imperfection is higher at the center of the cylinder, and this imperfection is defined as biased imperfection. We consider this imperfection as the most deleterious compared to the other geometric imperfections: local dimple imperfection, eigenmode imperfection, and unbiased sinusoidal axisymmetric imperfection for which the amplitude is constant along the length [18]. The mathematical expression of the biased imperfection is [18,21,22]

$$w = -R\left(a_{io} + a_i \cos \frac{x}{N_{\lambda}} \right) \frac{\cos \frac{\pi x}{\lambda}}{\lambda}$$

(6)

where $x$ is the axial coordinate with the origin placed at the center, $R$ is the radius of the cylinder, $a_{io}$ and $a_i$ are the relative value of unbiased and biased components of the amplitude, respectively ($R \times a_{io}$ and $R \times a_i$ are the absolute values of the unbiased and biased components), and $N_{\lambda} \times \lambda$ represents the length of the cylinder. The $\lambda$ represents the first eigenmode half wavelength of the circular cylinder under axial compression, i.e., $1.72\sqrt{Rt}$ for $\nu = 0.30$ [28]. In this study, the value of $a_{io}$ and $a_i$ is chosen such that the bias of the imperfection defined by the ratio $a_i/a_{io}$ is 5 [18,21,22]. We induce this biased geometric imperfection in both circular cylinders and wavy cylinders having $L_c = 3R/70$ m and $N = 15$. We find again that the wavy cylinder is insensitive to imperfection as in the case of axial compression.

Figure 11(a) shows the moment-rotation diagrams of the perfect and imperfect circular cylinders, while Fig. 11(b) shows the moment-rotation diagrams of wavy cylinders. The amplitude of imperfection is $t/10$ in both cases, where $t$ is the thickness of cylinders (0.0167 m). The presence of imperfection reduces the load carrying capacity of the circular cylinder significantly (around 15%), whereas the presence of imperfection does not reduce the capacity of wavy cylinder: the moment-rotation diagram of perfect and imperfect wavy cylinders is almost the same. It is astonishing that the shape and the amplitude of the imperfection are the same in both cases, but the response of circular and wavy cylinders is entirely different. In Fig. 12, the moment-rotation diagrams are shown for perfect and imperfect cylinders (circular and wavy) with higher amplitudes of imperfection, i.e., $5t/20$, $10t/20$, $15t/20$, and $20t/20$. The reduction in load carrying capacity is always more in case of circular cylinders, however, significant reduction also occurs for wavy cylinders with higher imperfection amplitude. This result is quite encouraging as the loading scenario, material properties, and the dimensions of the cylinder are different to the cylinders in the Sec. 3 and in the study of Ning and Pellegrino [15] showing constancy in the imperfection insensitivity.

In Fig. 13(a), the moment capacity of imperfect circular and wavy cylinders is plotted against the normalized imperfection amplitude, and in Fig. 13(b), the knockdown factor is plotted against the normalized imperfection amplitude. For small imperfection amplitude ($w_i < 0.3\nu$), the knockdown factor is almost unchanged for wavy cylinders, whereas for circular cylinders, it is reducing rapidly with increase in imperfection amplitude. The reduction in case of circular cylinders is around 35% for $w_i = 0.3\nu$, whereas for wavy cylinder, it is almost negligible. For higher imperfection amplitude ($w_i > 0.3\nu$), significant reduction is taking place for the wavy cylinders, but their knockdown factor is always higher than the knockdown factor of circular cylinders. Increasing the imperfection amplitude, the difference between knockdown factors for circular and wavy cylinders is reducing (Fig. 13(b)) and it is anticipated that for further increase in imperfection amplitude ($w_i > 2\nu$), the difference will be diminished. A first conclusion from these results is that wavy cylindrical shells are the most beneficial for small imperfection amplitudes which could be the case of new advanced manufacturing process. For higher imperfection amplitudes, the usefulness of using wavy cylinders diminishes, but the imperfections rarely reach that limit for all practical applications. Apart from higher knockdown factor, the wavy cylinders also have higher moment capacities (Fig. 13(a)). This is happening because the moment inertia of the wavy cross sections is higher than the moment of inertia of circular cross sections and also due to the stiffening effect of waviness.

It could be argued that the wavy cylinders require more material than the circular cylinders and this could reduce the associated benefits, i.e., imperfection insensitivity. To investigate this issue, we reduce the material volume required in the wavy cylinders, which can be done in two ways: first by reducing the thickness and keeping the base radius constant, and second by reducing radius keeping the thickness constant. Figures 14(a) and 14(b) show the

![Fig. 11](image-url)
moment capacities and knockdown factors, respectively, of the circular cylinder, and the wavy cylinders whose material volume is the same as the circular cylinder. For perfect cylinders, the capacity of the circular cylinder is higher than the capacity of both wavy cylinders (Fig. 14(a), moment corresponding to 0 imperfection amplitude). However, for all the cases of imperfect cylinders, the capacities of both wavy cylinders are higher than the circular cylinders. The knockdown factor of wavy cylinders, even after reducing its thickness or radius to make their volume equal to the circular cylinder, is much higher than the circular one. Therefore, wavy

Fig. 12 Moment-rotation diagram of perfect and imperfect circular and wavy cylinders. The reduction due to imperfection is always low for the wavy cylinders as compared to circular cylinders.

Fig. 13 (a) Moment capacities of the circular and the wavy cylinders against the imperfection amplitude. (b) Knockdown factor for the circular and the wavy cylinders against the imperfection amplitude. The moment capacity and knockdown factor of the wavy cylinders are always more than the moment capacity and knockdown factor of circular cylinders. In case of wavy cylinders, the knockdown factor is almost 1 (insensitive to imperfections) for imperfection amplitude $w_0 < 0.30t$.

Fig. 14 (a) Moment capacities and (b) knockdown factor of the circular and the two wavy cylinders. The material volume of the wavy cylinders is the same as the circular cylinder and is achieved by reducing the thickness for one case and by reducing the radius for the second case. The performance of wavy cylinders is still better than the circular cylinder.
cylinders are still preferable because their imperfection insensitivity surpasses the burden of extra material requirement. An interesting result emerges from Fig. 14(a): reducing thickness is a more beneficial choice than reducing the base radius for small imperfection amplitude, but for higher imperfection amplitude, reducing the base radius is preferable.

4.2 Effect of Wave Parameters on Knockdown Factors. In Sec. 4.1, a particular wavy cross section with \( A_r = 3R/70 \) m and \( N = 15 \) is used to demonstrate the benefits associated with wavy cylinders. To further explore the imperfection insensitivity of wavy cylinders, we created a total of 25 wavy cross sections. These 25 wavy cross sections are created using five wave amplitudes and five numbers of waves. The values of \( A_r \) are \( R/70 \) (1.7\( t \)), \( 2R/70 \) (3.4\( t \)), \( 3R/70 \) (5.1\( t \)), \( 4R/70 \) (6.9\( t \)), and \( 5R/70 \) (8.6\( t \)), and the values of \( N \) are 3, 5, 15, 20, and 25. All 25 cross-sections of the wavy cylinders are shown in Fig. 15. For imperfections, we are imposing the biased axisymmetric imperfection as given by Eq. (6).

First, we will discuss the effect of wave amplitude. The knockdown factor for the five cylinders with \( N = 15 \) and varying \( A_r \) are plotted against the normalized imperfection amplitude in Fig. 16. The knockdown factors are increasing with an increase in wave amplitude \( A_r \), i.e., for a given imperfection amplitude, the value of knockdown factor is higher for the cylinder having higher wave amplitude. These trends suggest that higher wave amplitudes improve the performance of wavy cylinders; they become less and less sensitive to imperfections. The difference among knockdown factors is more pronounced for small imperfection amplitudes, while for higher imperfection amplitudes, the difference among knockdown factors is reducing. These results signify the benefits of using wavy cylinders. Similar patterns, the increment in knockdown factor with the increment in \( A_r \), are observed for other number of waves, e.g., \( N = 3, 5, 20, \) and 25. Figure 17 shows the effect of wave amplitude \( A_r \) on the knockdown factor for \( N = 20 \) and \( N = 25 \). From these results, it can be concluded that the performance (insensitivity to imperfections) of wavy cylinders improves with the increase in the wave amplitude \( A_r \).

To show the effect of the number of waves \( N \), we keep \( A_r = 2R/70 \) (3.4\( t \)) and vary \( N \) from 0 to 25. The knockdown factors for these six cylinders are plotted against the normalized imperfection amplitude in Fig. 18. The effectiveness of wavy cylinders improves with an increase in the number of waves \( N \) along the circumference, i.e., wavy cylinders become less and less sensitive to imperfection as \( N \) increases. The number of waves plays a significant role on the performance of wavy cylinders, e.g., the reduction in load carrying capacity is 4% for \( N = 25 \) and 44% for \( N = 3 \), for imperfection amplitude 0.5\( t \). A similar pattern is obtained for the other wave amplitudes as shown in Fig. 19 for \( A_r = R/70 \) (1.7\( t \)) and \( A_r = 5R/70 \) (8.6\( t \)). There is not much difference between the response of circular cylinder and wavy cylinder with \( N = 3 \). But, the difference between circular and wavy cylinders becomes visible with higher numbers of waves, i.e., \( N = 5, 15, 20, 25 \). There are three notable characteristics coming out of Fig. 18: (1) the higher number of waves means better performance of wavy cylinders, (2) the differences among knockdown factors are more pronounced for small imperfection amplitude and diminishes with increase in imperfection amplitude, and (3) for small imperfection amplitude \((\delta t < 0.4t)\), the reduction in load carrying capacity is almost negligible for \( N = 20 \) and \( N = 25 \).

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Fig. 15 The cross sections of wavy cylinders used in this study. In total, five wave amplitudes \( A_r = R/70 \) m (1.7\( t \)), \( 2R/70 \) m (3.4\( t \)), \( 3R/70 \) m (5.1\( t \)), \( 4R/70 \) m (6.9\( t \)), and \( 5R/70 \) m (8.6\( t \)) and five numbers of waves \( N = 3, 5, 15, 20, \) and 25 are used to create the wavy cylinders. The radius of base circle \( R = 2 \) m and the radius to thickness ratio \( R/t = 120 \) apply for all cylinders.
The effectiveness of wavy cylinder is increasing with the increase of $A_r$. From the above parametric analysis, we have demonstrated that the wave parameters, $A_r$ and $N$, play a crucial role in the response of wavy cylinders to imperfections. Higher values of $A_r$ and $N$ lead to lower sensitivity to imperfections, with values of wave parameters that improve the knockdown factor substantially found around $A_r = 2R/70$ (5.1$t$) and $N = 15$.

**4.3 Effect of Orientation of Imperfections and Waves.** So far, the orientation of both the wave and the imperfection is concave at the meridian with maximum compressive stress at the middle of the cylinder under bending (case A of Figs. 20(a) and 20(b)) in anticipation that the maximum reduction in the moment capacity due to the imperfection will occur with this combination. But, the wave and the imperfection can have any orientation; Fig. 20 shows three orientations of both the wave and the imperfection (biased imperfection), which results in total nine core combinations of the wave and the imperfection. We analyzed the effect of imperfections in these nine combinations to investigate the importance of the orientation of the imperfection and the wave. The orientation is not an issue for axial compression as loading is symmetric but for bending, it might have some effect as the bending is asymmetric by nature.

We use the wavy cylinder with $A_r = 3R/70$ and $N = 15$ for this investigation. Figure 21 shows the knockdown factors for the nine combinations of the wave and the imperfection. Surprisingly, the orientation of the wave and the imperfection does not affect the knockdown factor significantly, as the nine curves in Fig. 21 are almost identical. This is significant and shows that wavy cylinders are indifferent to the orientation of the imperfection. This property is not available with conventionally stiffened cylinders, where if the stiffeners are not located in the middle of the compressive side, they would not be as effective under bending.

**4.4 Local Dimple Imperfections.** In all the previous subsections for the cylinders under bending, we have applied the biased geometric imperfection (Eq. (6)) as the most deleterious imperfection. To illustrate this, we use the dimple-like (Eq. (2)) imperfection and compare the response of wavy cylinders to these two imperfections: dimple-like and biased imperfection. The dimple-like imperfection is introduced on the meridian with maximum compressive stress at the middle of the cylinder under bending. Figure 22 shows the knockdown factor of wavy cylinders corresponding to biased axisymmetric and local dimple-like imperfections. For small imperfection amplitude ($w_0 < 0.4t$), both imperfections reduce the moment capacities almost equally, but for higher imperfection amplitudes ($w_0 > 0.4t$), the reduction due to the biased imperfection is more than the dimple-like imperfection and the difference between knockdown factor is increasing with imperfection amplitude. Wavy cylinders are therefore more effective when the imperfection is local in nature, which is a more realistic imperfection.

**4.5 Mass Efficiency and Comparison With Stiffened Cylinders.** Stiffeners are generally used to increase the load carrying capacity of thin cylindrical shells and to reduce their sensitivity to imperfections [14]. The wavy cylinders can also be assumed as a kind of stiffened cylinders. It is of interest to compare the imperfection sensitivity of conventionally stiffened cylinders and wavy cylinders. For this purpose, one circular, one wavy (with $A_r = 3R/70$ and $N = 15$), and four stiffened cylinders are created as shown in Fig. 23. Four longitudinal stiffeners along the circumference are used for stiffened cylinders 1 and 2, and 16 longitudinal stiffeners along the circumference are used for stiffened cylinders 3 and 4. For stiffened cylinder 1, the width (circumferential direction) of stiffeners is $5t$ and the depth (radial direction) of stiffeners is $32t$, while for stiffened cylinder 2, the width of stiffeners is $5t$ and the depth of stiffeners is $72t$, where $t$ is the thickness of cylinders. Similarly, for stiffened cylinder 3, the width of stiffeners is $5t$ and the depth of stiffeners is $2t$, while for stiffened cylinder 4, the width of stiffeners is $5t$ and the depth of stiffeners is $4t$. The volume of material required for the circular, wavy, stiffened 1, stiffened 2, stiffened 3, and stiffened 4 cylinders are $V_0$, $1.1V_0$, $1.2V_0$, $1.4V_0$, $1.2V_0$, and $1.4V_0$, respectively, where $V_0$ represents the volume of the circular cylinder.
To study the effect of imperfections, we used the biased imperfection (Eq. (6)). The moment capacities and knockdown factors of these six cylinders are shown in Figs. 24(a) and 24(b), respectively. Stiffeners not only increase the bending capacity of the circular cylinder but also reduce the sensitivity of the circular cylinders to imperfections. When compared to wavy cylinders, the knockdown factors for the wavy cylinder are higher than the four stiffened cylinders for imperfection amplitude \( w_o < 0.75t \). Although the capacity of the perfect stiffened cylinders is more than the perfect wavy cylinder, for imperfect cylinders, the capacity of the wavy cylinder is more than the stiffened cylinder 1 for \( 0.05r < w_o < 0.75r \) and more than stiffened cylinders 2 and 3 for \( 0.2r < w_o < 0.55r \). Stiffened cylinder 4 always has more capacity than the wavy cylinder. An important feature of the wavy cylinder is the imperfection insensitivity for imperfection amplitude \( w_o < 0.4t \) (i.e., knockdown factor is almost (1), when compared to all the alternatives in this analysis). This reduces the uncertainties associated with all practical (imperfect) cylinders which does not apply for the stiffened cylinders. In addition, the material required for the wavy cylinder is 10% more than the circular cylinder, while the material required for the stiffened 1, stiffened 2, stiffened 3, and stiffened 4 cylinders are 20%, 40%, 20%, and 40% more than the circular cylinder, respectively. Wavy cylinders are not only performing better than the stiffened cylinders considered in this study, in terms of imperfection insensitivity for small imperfection amplitude \( w_o < 0.75r \) but are also more economical in terms of required material. An interesting finding is that the many small distributed stiffeners are more beneficial in terms of bending capacity compared to wavy cylinders as waviness is present across the circumference. In other words, waviness acts as a stiffener but distributed uniformly across the circumference.

5 Conclusions

For the wavy cylinders under axial compression, we found that dimple-like imperfection does not reduce the load carrying capacity significantly in the range of imperfection amplitude considered in this study. This is aligned with the findings of Ning and Pellegrino [14,15], although they considered eigenmode imperfections. Many factors are responsible for making wavy cylinders insensitive to imperfections: (1) reduction in effective radius of curvature [14], (2) stiffness introduced by waviness, and (3) periodic change in...
the direction of curvature that creates disharmony with imperfection and consequently reducing the impact of imperfections. These factors are engaged simultaneously to achieve the imperfection insensitivity. We did not delve further in evaluating separate contributions of these three factors, but instead we studied the effectiveness of wavy cylinders under bending—an important loading scenario for many applications of thin cylinders, considering inelastic behavior.

For the wavy cylinders under bending, we again found that wavy cylindrical shells are insensitive to the imperfections and the presence of imperfections does not reduce the load carrying capacity significantly. Previous studies [14,15] have revealed similar conclusions for thin cylindrical shells under axial compression. Based on the comparisons between the knockdown factors of the wavy cylinders and the circular cylinders, we report that for small amplitudes of imperfection ($w_0 < 0.3t$), the reduction in bending capacities is high for the circular cylinders compared to the insignificant reduction for the wavy cylinders. Apart from this, we also found that the bending capacities of wavy cylinders are higher than the bending capacities of the circular cylinders. The wave parameters, i.e.,
Calladine, C. R., 1989, and in the future, the other territories of thin wavy cylindrical study explores an important aspect of thin wavy cylindrical shells with circular cylinders has not been done. Nevertheless, this speci-
cross-sectional thin cylindrical shells has not been studied, the prac-
tical applications of wavy cross-sectional thin cylindrical shells can be circumvented and material can be used optimally. This study is limited in some respects: the manufacturing feasibility of wavy cross-sectional thin cylindrical shells has not been studied, the imperfection sensitivity) of thin circular cylindrical shells can be increased. For the higher wave amplitudes and higher number of waves, this requires.

These results are very promising as the imperfection sensitivity of thin cylindrical shells has been a big obstacle for their economic applications for a long time. This study shows that if wavy cross-
sectional cylindrical shells are used, the inherent drawback (high imperfection sensitivity) of thin circular cylindrical shells can be circumvented and material can be used optimally. This study is limited in some respects: the manufacturing feasibility of wavy cross-sectional thin cylindrical shells has not been studied, the practical applications of wavy cross-sectional thin cylindrical shells has not been considered, the specific R/t ratio (120) is used, and the cost comparison of wavy cylinders with circular cylinders has not been done. Nevertheless, this study explores an important aspect of thin wavy cylindrical shells and in the future, the other territories of thin wavy cylindrical shells can be explored.

References