Instability of thin steel cylindrical shells under bending

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1. Introduction

Among the many applications of steel cylindrical shells, wind turbines are one with continuously increasing use and it is anticipated that this trend will accelerate in the coming future. The need for more vigorous wind harvesting has given impetus not only to many technological advances but also to the design of more efficient wind turbines. One of the many technical goals of today’s wind energy industry is to develop solutions for taller wind turbine towers because the increase in their height is imperative to achieve more efficiency as the wind profile is stronger and more stable at higher elevations \cite{1}. However, tall and super tall wind turbine towers pose numerous design challenges to structural engineers. Tall wind turbine towers are generally made of thin steel cylindrical shells with the radius-to-thickness ratios \(R/t\) varying from 60 to 120 \cite{2–5}. The primary load on the towers is bending and thus understanding the bending behavior of thin steel cylindrical shells within this range of slenderness is essential to the design challenges.

The phenomenon of buckling of cylindrical shells under bending has been among the most interesting instability mechanics problems. It is a complex phenomenon due to the interaction of two different modes of failure, namely, ovalization and bifurcation. Ovalization is the phenomenon in which the cross-section of the cylindrical shell changes from circle to oval under the application of bending moment \cite{6}. The primary consequence of ovalization is the non-linear moment-curvature relationship that is represented as:

\[ M = EI \left( M \right) \frac{du}{dx} ^2 \]

where \(M\) is the applied bending moment, \(I\) is the moment of inertia, \(E\) is Young’s modulus and \(\frac{du}{dx} \) is the curvature. Due to the non-linearity of Eq. (1), there exists a curvature \(\frac{du}{dx}\) at which the moment reaches its peak value and starts reducing even when the curvature increases. This peak moment is known as the limit moment, and it could happen in elastic or inelastic range depending on the value of \(R/t\) and the material properties. Brazier \cite{6} derived the equation for the limit moment in the elastic range that is expressed as:

\[ M_{\text{limit}} = 2 \sqrt{2} \frac{E \pi R^3}{9} \]

Many researchers studied the elastic ovalization of cylindrical shells under different loading conditions using the refined shell theory \cite{7–13}. Later, the study of inelastic ovalization was motivated by offshore gas pipelines, which always fail due to inelastic buckling \cite{2}. The
group of Kyriakides [14–16] and Karamanos and Tassoulas [17] studied the inelastic ovalization under combined bending and external pressure. All the above-mentioned studies are related to one aspect of thin cylindrical shells under bending, i.e., ovalization. Bifurcation instability and its interaction with ovalization are other important aspects of the phenomenon. This prebuckling nonlinear ovalization phenomenon is the main difference between thin cylindrical shells under pure bending and thin cylindrical shells under pure compression. For cylindrical shells under pure axial compression, buckling appears in the form of wrinkles when stresses reach a critical stress $\sigma_{cr}$. The value of $\sigma_{cr}$ is [18]:

$$\sigma_{cr} = \frac{E}{\sqrt{3(1 - \nu^2)}} \left( \frac{t}{R} \right)$$

(3)

Neglecting imperfection effects, this equation can only be applied for very short cylindrical shells under bending, for which the ovalization is constrained by the end boundary conditions [19]. Ovalization can also be prevented for longer cylinders by stiffening techniques such as stiffening rings. For these cases, the critical moment can be defined as:

$$M_{\text{bifurcation}} = \frac{E \pi R t^2}{\sqrt{3(1 - \nu^2)}}$$

(4)

where $\nu$ is the Poisson's ratio. Eqs. (3) and (4) are valid only if the cross-section of the cylinder does not change, i.e., ovalization is prevented. If ovalization is allowed, the interaction between ovalization and bifurcation takes place and the value of $\sigma_{cr}$, given by Axelrad [20], is:

$$\sigma_{cr} = \frac{E}{\sqrt{3(1 - \nu^2)}} \left( \frac{t}{R_{\text{local}}} \right)$$

(5)

where $R_{\text{local}}$ is the local radius at the ends of the minor axis of the ovalized cross-section, at the region of maximum curvature (compressive side). It is essentially the same formula that Hutchinson [21] found for the elastic buckling of oval shape cylinder under axial compression. The value of $R_{\text{local}}$ is always more than $R$ and thus due to pre-buckling ovalization, the moment required for bifurcation is reduced. The bending capacity of cylindrical shells in the elastic range is the smallest of the two: $M_{\text{bifurcation}}$ and $M_{\text{local}}$. Numerous studies aimed to understand the interaction between ovalization and bifurcation in the elastic range and varied loading conditions [22–31]. The interaction of ovalization and bifurcation is more complex in the inelastic range. Failure could occur due to the ovalization or bifurcation depending on the $R/t$ ratio and material properties. Studies had been done to understand the interaction of ovalization and bifurcation in inelastic range [32–38,19], but for lower $R/t$ ratio ($<60$).

The interest in the bending behavior of thin steel cylindrical shells, with higher $R/t$, has been increased recently due to the demand of tall wind turbine towers, tubular pipes etc [2,39,4,34–44]. The dimensions of the cylindrical shells used in these applications are different to gas pipelines and thus new studies are needed focusing on higher $R/t$. Although with these studies, the progress has been made to understand the bending behavior of thin cylindrical shells under bending, which are used in tall and super tall wind turbine towers. But, these studies do not attempt to understand, thoroughly, the effect of material hardening models on the bending capacity. This aspect is crucial as these cylinders fail inelastically, i.e., at the peak bending moment, the stresses at some part of cylinders are more than yield stress limit. The part of this study is an attempt to fill this gap.

A hugely important characteristic of the instability of thin cylindrical shells stems from their imperfection sensitivity. Cylindrical shells are highly sensitive to imperfections and the presence of imperfections can reduce the load carrying capacity disproportionally to the size of imperfections. Koiter [45] demonstrated the reason of imperfection sensitivity of thin cylindrical shells in his celebrated thesis. Extensive study had been done to understand the imperfection sensitivity of thin cylindrical shells [46,47,2,19,48–50]. The current practice to deal with the imperfection sensitivity of cylindrical shells is to use conservative knockdown factors, which have been developed by NASA in the late sixties. The capacity of perfect cylinders is reduced by the knockdown factor and assigned as the design capacity. The values for the knockdown factor are provided by NASA [51] and shown in Fig. 1. To tackle the imperfection sensitivity, Eurocode [52] recommends inducing equivalent geometric imperfections in the perfect cylinder, the amplitude of which is specified based on the fabrication quality. The sensitivity of thin cylindrical shells depends on many factors, i.e., $R/t$ ratio, material properties, loading scenarios, type and amplitude of imperfection. Thus knockdown factor must take account to these parameters, which is not the case with current design practices. Here, we attempt to find a rational knockdown factors, which could be applied to thin cylindrical shells (used in tall wind turbines towers) under bending.

![Fig. 1. Distribution of experimental value of knockdown factor against the R/t ratio along with NASA [51] recommendation for lower bound knockdown factor. Reproduced from the report of Seide et al. [65].](image-url)
phenomenon. In Fig. 2, a typical example of a moment-curvature diagram and the evolution of ovalization of the cross-section at the middle of the cylinder is shown for a thin steel cylindrical shell under bending having an axisymmetric sinusoidal biased geometric imperfection (biased imperfection has varying amplitude along the longitudinal direction, see Section 3.3.4 for detail). One end of the cylinder is fixed and rotation is applied to the other end following a displacement-based FEM analysis. The curvature is found by dividing the end rotation, by the initial length of the cylinder. The moment is normalized by \( M_0 = D_n t_0 \), and the curvature is normalized by \( \kappa_i = \frac{1}{D_m} \), following the example of Vasilikis et al. [2]. \( D_m \) is the mean diameter of the cylinder (i.e., \( D - t \), where \( D \) is the outer diameter of the cylinder).

Initially, the moment-curvature diagram (Fig. 2a) is linear, subsequently becomes nonlinear, and finally, buckling takes place. It is of interest to analyze the moment-curvature diagram closely along with the ovalization of the cross-section at the mid-length of the cylinder. Fig. 2 includes 4 regions on the basis of the relation between the moment and the curvature. The moment-curvature relationship is linear in the first region where the normalized curvature varies from 0.00 to 0.29. This region ends with the appearance of nonlinearity in the moment-curvature relationship. In this region, the ovalization is insignificant as can be seen in Fig. 2b and the stresses, throughout the cylinder, are below the yield stress limit. Local stresses are less than the yield stress limit and the moment is below the yield moment. Thus, the classical moment-curvature relationship (\( M = \frac{E t^3}{3 D} \kappa \)) is valid in the first region as \( I \) can be assumed constant. In region two, where the normalized curvature lies between 0.29 to 0.40, ovalization becomes visible (Fig. 2b) and thus moment-curvature relationship becomes nonlinear although the stresses, throughout the cylinder, are still below the yield stress limit. This non-linearity comes from the ovalization and therefore the classical moment-curvature relationship is not valid in this region. At normalized curvature 0.40, the first yielding occurs (i.e., stresses cross the yield stress limit locally, but the moment is less than the yield moment) and material non-linearity starts playing its role. Material non-linearity increases the rate of ovalization (Fig. 2b), which is manifested as a highly nonlinear moment-curvature relationship. At normalized curvature 0.80, shell buckling takes place and the bending moment starts reducing. After the point of buckling, the cross-section changes very rapidly as can be seen in Fig. 2b.

To understand the phenomenon in detail, 10 points are identified in the moment-curvature diagram as shown in Fig. 3. These points represent 10 stages of the applied rotation. The deformation pattern for these 10 stages included in Fig. 3 shows that buckling localizes eventually. At stage 1, no rotation is applied and thus line is in the initial shape. It can be seen that the line is not exactly straight due to the initial geometric imperfection in the form of a sinusoidal wave. At stage 2 and 3, there is not any significant change in the shape of this line although it is rotated due to the bending. At stage 4, first yielding occurs and a small change takes place in the shape of the line. At stage 5, the change in the shape of the line is significant and the sinusoidal pattern can be observed. This is a clear sign of the wrinkling which happens near stage 5. At stage 6, the change in the shape of the line is increased and at stage 7, buckling takes place. After buckling, the deformation is concentrated only at the center of the line as shown in Fig. 3 (stages 8, 9 and 10). It can also be seen that the wrinkling disappears after buckling. Wrinkling takes place in the inelastic range before buckling, and disappears after buckling, as the stresses concentrate in the middle of the cylinder.

To measure the appearance of wrinkling quantitatively, the variation of the amplitude of imperfection against the curvature is studied. The amplitude of the imposed geometric imperfection is \( t/10 \) and this varies as the rotation is applied. To measure the amplitude of wrinkling, three nodes (A, B and C) in the middle of the cylinder are selected (Fig. 4). A and B are at the crest and C is at the trough. The displacement of nodes A, B and C are then traced relative to the applied rotation and these displacements are used to find the amplitude of imperfection plus wrinkling. First, the relative displacement vector of node A and B with respect to node C is found, i.e., vectors \( v_1 \) and \( v_2 \) (Fig. 4). Using these vectors, the angle \( \phi \) is obtained. The amplitude of imperfection plus wrinkling will be half of the projection of vector \( v_1 \) on the line that is normal to the line AB. The expression for \( w_0 \) can be found using the expressions:
This formulation is true only if the nodes A, B, and C are in the plane of bending. The evolution of \( w_o \) at every step is obtained using the method described above and \( w_o \) is plotted against the applied curvature (Fig. 4). Initially, \( w_o \) remains approximately \( t/10 \), which is expected as the applied rotation is small. It must be noted that \( t/10 \) is the amplitude of the geometric imperfection and therefore the starting point of the figure. The significant increment in \( w_o \) occurs at around 0.50 normalized curvature, which means that wrinkling is happening. Finally, wrinkling in the form of buckling appears at 0.80 normalized curvature. From these observations, it can be observed that the presence of imperfections triggers wrinkling, which finally manifests in the form of buckling.

Although the ovalization in the middle of the cylinder is more pronounced (Fig. 2b), it is interesting to see in further detail the ovalization at different locations of the cylinder. Fig. 5 presents the ovalization for a middle part of the cylinder with a length \( 5\lambda \) (\( \lambda \) here is the half-wavelength of the elastic buckling eigenmode by Timoshenko and Gere [53]). Initially, the ovalization is small and uniform through the whole \( 5\lambda \) section. But just before buckling (at normalized curvature 0.75 – 0.89), nonuniform ovalization occurs (Fig. 5), which implies the wrinkling phenomenon. Finally, buckling takes place at 0.80 normalized curvature and the sharp increase can be seen at this curvature in Fig. 5 throughout the section.

The stress distributions at the 10 stages of bending are shown in Fig. 6a. Stage 1 represents the initial no-stress state. At stage 2, some rotation is applied and thus a nonzero stress-distribution can be seen. It can also be seen that the distribution of stresses follows a wavy shape at stage 2. This is happening because the cylinder is imperfect and the shape of imperfection is sinusoidal. The stresses are intensified at stage 3 as the applied rotation is increased. The classical moment-curvature relationship is valid for stage 2 and 3 because the stresses are less than their yield stress limit and the ovalization is very small. First yielding
occurs at stage 4 and the region of plasticity increases at stage 5 and 6. Inelastic localized buckling takes place at stage 7. After the buckling, the bending moment starts dropping rapidly, which leads to stress redistribution. This redistribution affects the center region and the edge regions of the cylinder differently. In the center region, stresses are almost constant (Fig. 6b), while in edge regions, stresses are reducing rapidly (Fig. 6c) after buckling. These are typical characteristics of a local shell buckling phenomenon. In this case, the cylinder fails due to the shell buckling of the center region, and thus this region follows its post-buckling path. In this region, stresses are not reducing in the post-buckling range. The reduction in the bending moment is compensated by the change of the cross-section, which consequently reduces the moment of inertia. The regions beyond the center do not buckle and they follow an unloading path with the reduction of the bending moment. With this background of bending instability of thin steel cylindrical shells, the effect of geometric imperfections and material hardening models on the bending behavior of cylindrical shells is further presented.

3. Finite element modeling

This section describes geometry, boundary conditions, strain-hardening models and Finite Element Modeling of the cylindrical shells used in this study along with the description of geometric imperfections used.
to make imperfect cylinders.

3.1. Geometry of the model

A 20.00 m long steel cylindrical shell with 2.00 m diameter is chosen for the analyses. The thickness \( t \) of the cylindrical shells is chosen so that the ratio \( \frac{t}{R} \) varies from 60 to 120 with an interval of 20. The reason behind choosing these dimensions is that they cover a section of tall and super-tall wind turbine towers. For the analysis purposes, a displacement method of analysis is used with one end of the cylinder is fixed while the rotation is applied at the other end as shown in Fig. 2. The simulation is performed in ABAQUS [54] utilizing the Riks method and using four-node reduced integration shell (S4R) elements. Four integration points are utilized along the thickness of each element. Two rigid body constraints are imposed at the end cross-sections, which make sure that the end cross-sections do not change their shape, i.e., ovalization is prevented during the analysis. These constraints represent the rigid rings, which are used in cylindrical shells at regular intervals to prevent the ovalization [55]. After performing an extensive mesh convergence analysis, it was found that around 20,000 elements (each with a dimension of 121.20 mm \( \times \) 121.20 mm) provide sufficient accuracy for all the \( \frac{R}{t} \) ratios and the material-hardening models (see next section for material hardening models). In Fig. 7, the results of mesh convergence analysis are shown for \( \frac{R}{t} \) ratios 60, 80, 100 and 120 and material hardening models \( n = 9 \), \( n = 13 \) and \( n = 30 \) (see the next section for the meaning of \( n \)). The results are shown in terms of normalized moment capacity and \( a/\sqrt{Rt} \), where \( a \) is the element size (elements are square in shape), \( t \) is the thickness of the cylinders and \( R \) is the radius. For 20,000 elements, the values of \( a/\sqrt{Rt} \) are 0.43, 0.50, 0.56 and 0.61 for \( \frac{R}{t} \) ratios 60, 80, 100 and 120 respectively. These values are quite smaller than the linear bending half-wavelength (i.e., \( 2.44(\sqrt{Rt}) \)) and the classical axisymmetric buckle half-wavelength for cylindrical shells under axial compression [53] and thus the discretization is considered adequate.

![Fig. 7. Mesh convergence analysis, around 20,000 S4R elements give sufficiently accurate results. (a) \( R/t = 60 \), (b) \( R/t = 80 \), (c) \( R/t = 100 \) and (d) \( R/t = 120 \). The mesh sizes corresponds to 20,000 elements are 0.43, \( \sqrt{Rt} \), 0.50, \( \sqrt{Rt} \), 0.56, \( \sqrt{Rt} \) and 0.61, \( \sqrt{Rt} \) for \( R/t = 60 \), \( R/t = 80 \), \( R/t = 100 \), and \( R/t = 120 \) respectively. The element sizes are small than the first mode half-wavelength and thus able to capture the mode of failure.](image)

![Fig. 8. Stress-strain diagrams for the bilinear and three versions of Ramberg-Osgood plasticity model.](image)
3.2. Strain-hardening model

Thin steel cylindrical shells in the range $60 < R/t < 120$ are expected to buckle inelastically and therefore the strain-hardening model plays a highly important role in their behavior. To understand the role of strain-hardening models in the bending behavior of thin steel cylindrical shells, four kinds of strain-hardening models—a bilinear elastic-perfectly plastic and three versions of the Ramberg-Osgood plasticity model—are used, as shown in Fig. 8. The Ramberg-Osgood stress-strain relationship used in this study is (Kyriakides and Corona [56]):

$$\varepsilon = \frac{\sigma}{E} \left[ 1 + \frac{3}{7} \left( \frac{\sigma}{\sigma_y} \right)^{n-1} \right]$$

(9)

The value for Young modulus $E$ is 210.00 Gpa, the Poisson’s ratio $\nu$ is 0.30 and the yield stress $\sigma_y$ is 355.00 Mpa. Three values of $n$, i.e. 9, 13 and 30, are considered in this study.

3.3. Description of geometric imperfections

To study the effect of geometric imperfections on the bending behavior of thin steel cylindrical shells, four kinds of geometric imperfections are introduced in-turn in the perfect cylinders.

3.3.1. Modal shape imperfection

The first geometric imperfection is the first Modal shape of the cylindrical shell under bending, which is obtained from the eigenvalue analysis of the perfect cylindrical shell under bending (Fig. 9). Modal shape imperfection cannot be expressed by a simple analytical expression as it is obtained through the Finite Element Analysis. To create the Modal shape imperfect cylinder, a scaled Modal shape was introduced to the perfect cylinder having a pre-defined amplitude using the scalar $w$ as shown in Fig. 9.

3.3.2. Dimplelike imperfection

The second geometric imperfection used in the analysis is the Dimplelike imperfection, which is localized at a particular region. This type of imperfection is considered realistic in thin steel cylindrical shells, when some solid hits the cylindrical shell during transportation or construction (Fig. 10). Different mathematical equations have been proposed to describe these dimples [57,58]. The present study uses the equation of dimple proposed by Hutchinson [58] for the spherical shell, which is modified to suit the cylindrical shell. The mathematical equation of Dimplelike imperfection for the cylindrical shell is:

$$w = -w_{oi} e^{-\frac{x^2}{(\frac{R}{2})^2}} e^{-\frac{(\frac{R}{2})^2}{\lambda^2}}$$

(10)

where $w$ represents the deviation from the original position in the radial direction and $w_{oi}$ is the amplitude of the imperfection; $x$ and $\theta$ are the axial and circumferential coordinates with the origin placed in the middle of the cylinder. $L_1$ and $\theta_1$ are the parameters which decide the length (in axial direction) and width (in circumferential direction) of the dimple. The values for $L_1$ and $\theta_1$ are chosen such that the length ($2L_1$) and the width ($2\theta_1$) of the dimple are equal to the first mode-shape wavelength of the cylindrical shell under compressive load, i.e. $3.44\sqrt{Rt}$ for $\nu = 0.30$ [53]. The value of half-wavelength of the mode-shape is represented by $\lambda$. The value of half-wavelength $\lambda$ in this study is taken as $1.72\sqrt{Rt}$, which is classical axisymmetric buckle half-wavelength for cylindrical shells under axial compression [53]. This value is less than the linear bending half-wavelength of cylinders under bending ($2.44\sqrt{Rt}$). We calculated the half-wavelength of the first eigenmode for the cylinders (used in this study) under bending and found that they vary from $1.76\sqrt{Rt}$ to $1.96\sqrt{Rt}$. And thus, we chose a smaller value, i.e. $1.72\sqrt{Rt}$, so that the analysis can capture buckling and post-buckling behavior accurately.

$$\theta_1 = 1.72\sqrt{Rt}$$

(11)

$$L_1 = 1.72\sqrt{Rt}$$

(12)

Fig. 10. The induction of Dimplelike geometric imperfection.

3.3.3. Axisymmetric harmonic imperfection with constant amplitude along the longitudinal direction (unbiased imperfections)

The two geometric imperfections mentioned above are non-axisymmetric. The present work also considers two axisymmetric geometric imperfections. The first one is in the shape of the sinusoidal wave whose half-wavelength is $\lambda$ (where $\lambda$ is the half-wavelength of the cylindrical shell under compressive load [53] i.e. $1.72\sqrt{Rt}$). In this paper, this imperfection is defined as unbiased imperfection, and mathematically this imperfection can be described as:

$$w = -R\omega_0 \cos \left( \frac{\pi x}{L} \right)$$

(13)

where $w$ represents the deviation from the original position in the radial direction, $L$ is the length of the cylinder, $R\omega_0$ is the amplitude of imperfection and $x$ is the axial coordinate with the origin placed in the middle of the cylinder. This equation is generated by modifying the biased imperfection given by Ju and Kyriakides [36]. The perfect and imperfect cylinder are shown in Fig. 12 along with the variation of $w$ along the length.
3.3.4. Axisymmetric harmonic imperfection with varying amplitude along the longitudinal direction (biased imperfections)

This imperfection is similar to the unbiased imperfection with a slight modification. In this case, the amplitude is slightly higher at the center as shown in Fig. 13. This imperfection is defined as biased imperfection as its amplitude is higher at the center. Its mathematical expression is [56,36]:

$$w = (-R)(a_d + a_i \cos \frac{\pi x}{N\lambda}) \cos \frac{\pi x}{\lambda}$$

where, $x$ is the axial coordinate with the origin placed at center, $R_{a_d}$ and $R_{a_i}$ are the unbiased and biased components of amplitude respectively and $N\lambda$ represents the length of the cylinder. In this study, the value of $a_d$ and $a_i$ are chosen such that the ratio $\frac{a_i}{a_d}$ is 5. The variation of the amplitude along the length is depicted in Fig. 13. The rationale behind using biased and unbiased imperfections is that they are generally more deleterious than the Dimplelike imperfection and Modal shape imperfections. Apart from that, these imperfections (biased and unbiased) are used in past studies of cylindrical shells under bending [56,36].

4. Results from computational analysis

4.1. Effect of geometric imperfections

To study the effect of geometric imperfections on the bending behavior of thin steel cylindrical shells, four geometric imperfections are introduced in the perfect cylinders in-turn. The amplitude of imperfections is kept to $t/10$ for all the cases. The first results are presented in Fig. 14 for $R/t = 60$ and Ramberg-Osgood strain-hardening model with $n = 9$. It can be seen that the thin steel cylindrical shells are highly imperfection sensitive under bending in the inelastic range. For the Modal shape imperfect cylinder, the reduction in peak moment and collapse curvature are about 14% and 50%, respectively. These reductions are significant compared to the amplitude of imperfection, i.e., $t/10$. Surprisingly, the reduction in the collapse curvature is much higher than the reduction in peak moment. This could be explained by the existence of a highly nonlinear relationship between the moment and curvature before the buckling and thus for a small reduction in the moment, a large reduction in the curvature is required. The large reduction in collapse curvature signifies that the presence of imperfects is more critical when the displacement is the design criteria. The presence of imperfections also changed the location of the first yielding in the moment-curvature space. The first yielding occurs at a lower moment of the imperfect cylinder, which can be attributed to the stress concentration in the imperfect cylinders. The pre-buckling path of perfect and Modal shape imperfect cylinders is almost same as shown in Fig. 14a.

The impact of Dimplelike imperfection is least deleterious among the four kinds of imperfections considered in the analysis. The reduction in the peak moment and the collapse curvature are approximately 6% and 30%, respectively. Here also, the reduction in the collapse curvature is much more than the reduction in the peak moment. In the case of Dimplelike imperfection, the location of first yielding, in moment-curvature space, for the perfect and the imperfect cylinders are almost same (Fig. 14b). The pre-buckling path of the Dimplelike imperfect cylinder and the perfect cylinder is approximately the same.

The impact of the biased imperfection is the most severe among the four geometric imperfections considered (Fig. 14c). The reduction in the peak moment and the collapse curvature due to the biased...
imperfection are, 18% and 51%, respectively. For the unbiased imperfect cylinder, the reduction in the peak moment and collapse curvature are, 17% and 49%, respectively (Fig. 14d). The pre-buckling path of the perfect and the imperfect cylinders are different in the cases of biased and unbiased imperfection in contrast to the Modal shape imperfection and the Dimplelike imperfection where the pre-buckling path of perfect and imperfect cylinders are almost the same. From these analyses, it can be concluded that thin steel cylindrical shells are very sensitive to the imperfections under inelastic bending ($60 < R/t < 120$).

It is noteworthy to see the impact of shell slenderness ($R/t$) on the imperfection sensitivity of thin steel cylindrical shells. Fig. 15 shows the variation of load reduction factor $\alpha$ and collapse curvature reduction factor $\Phi$ with $R/t$, respectively. From Fig. 15, it can be seen that the Dimplelike imperfection is least catastrophic and the biased imperfection is the most catastrophic among the four geometric imperfection shapes. Another important observation from Fig. 15 is that $\alpha$ and $\Phi$ do not vary significantly with the $R/t$, which is expected since the size and the amplitude of imperfections are scaled with $R/t$ (the amplitude is always $t/10$ and for this parametric analysis, $R = 200 m$ and $t$ varies so that $60 < R/t < 120$). The amplitude and the size of imperfections for $R/t = 60$ are higher than for $R/t = 120$, which makes $\alpha$ and $\Phi$ relatively independent from the $R/t$ ratio.

### 4.2. Effect of imperfections amplitude

In the previous section, the amplitude of imperfections is kept constant, i.e., $t/10$. In reality, the amplitude of imperfections varies significantly depending on manufacturing processes, transportation, and assembly methods (for wind turbine towers). It is interesting to study the role played by the imperfection amplitude on the bending

![Fig. 13. Mathematical description of the biased imperfection along with a perfect and a biased imperfect cylinder. The variation of imperfection amplitude along the length is also shown.](image)

![Fig. 14. Effect of (a) Modal shape imperfection, (b) Dimplelike imperfection, (c) unbiased imperfection and (d) biased imperfection on the bending behavior of thin steel cylindrical shells. The moment is normalized by $M_p = D_m t^2$ and the curvature is normalized by $K_i = \frac{1}{E I}$](image)
behavior of thin steel cylindrical shells.

For this study, the Dimplelike imperfection was selected, as it is more realistically present (created when some solid hits the cylinder during transportation or construction) in the cylinders as compared to other types of imperfections considered in this study. Another realistic and commonly studied imperfection is weld depression [59], but that is not considered in this study as we are not considering the effect of welding, i.e., geometric imperfections and residual stresses. The length and the width of the dimple are the same as in the previous section, i.e. \( \lambda_2 \) (Fig. 11), \( R/t \) ratio is 60 and the strain-hardening model is Ramberg-Osgood with \( n = 9 \). The amplitude of dimple \( w_0 \) is varied from 0 to \( t/2 \) with an interval of \( t/20 \) and a perfect and 40 imperfect cylinders have been analyzed. Fig. 16a shows the bending response of these 41 thin steel cylindrical shells. Many significant observations could be made from Fig. 16a:

1. The load carrying capacity of cylindrical shells reduces with the increase of imperfection amplitude until a certain limit and stabilizes with further increase of the imperfection amplitude. This is shown more clearly in Fig. 16b where the variation of moment capacity is plotted with the amplitude of imperfection. One can observe the existence of a plateau in Fig. 16b, which signifies that the moment capacity of imperfect thin steel cylindrical shells are independent of the amplitude of imperfection once the amplitude
crosses a specific limit (in this case for \( w_0 > 1.30\)). Vasilikis et al. [2] have investigated the effect of imperfection amplitude on the bending capacity of thin cylindrical shells, but they did not increase the imperfection amplitude more than 1.14\( t \). For that reason, the full length of the plateaus was not shown in their moment capacity vs imperfection amplitude plot. This finding is significant to show that thin cylinders possess a minimum capacity irrespective of imperfection amplitudes. It is surprising to compare this finding with the results of Hutchinson [58] and Lee et al. [60] who also found the existence of a plateau but for the spherical shells under uniform external pressure in the elastic range. This unexpected similarity of results between two different structures and loading scenarios, the elastic spherical shells under uniform external pressure and the bending of inelastic thin cylindrical shells, might be attributed to the unifying thin shell characteristics in both cases, i.e., the sensitivity to imperfections.

2. The other important characteristic, which emerges from Fig. 16a, is the diminishing of the peak in the moment-curvature diagram with the increase of the imperfection amplitude. For the high imperfection amplitude (\( w_0 > 1.30\)), there is no visible sharp peak in the moment. The moment increases with the increase in the curvature and reaches a limiting value (for \( w_0 > 1.30\)), remaining almost constant at this limiting moment even when the curvature is increasing and finally, collapse takes place. For the small imperfection amplitude (\( w_0 < 1.30\)), there exists a visible peak moment and collapse takes place at this peak moment. These differences can be explained by the extent of yielding in the two cases. For small imperfection amplitudes (\( w_0 < 1.30\)), prebuckling stresses cross the yield stress limit in a very small region in the middle of the cylinder. The prebuckling bending stiffness does not reduce significantly and consequently there is no large prebuckling rotation. For high imperfection amplitudes (\( w_0 > 1.30\)), prebuckling stresses cross the yield stress limit at a large region in the middle of the cylinder. As a result, a significant reduction occurred in the bending stiffness of the cylinder. This leads to large prebuckling rotation. The plateau can be seen in the moment-curvature diagram. This is an important finding, which shows that same imperfection shapes with different amplitude causes different modes of instability. Another observation is that cylinders with higher imperfection amplitude buckle smoothly, while cylinders with small imperfection amplitude fail abruptly. These distinct features of the moment-curvature diagram for the different range of imperfection amplitudes can be compared with the load-deformation curve of Eurocode [52] as given in its figure 8.6. Eurocode [52] also provides a criterion to calculate the load capacity (section 8.7.2.6) for these different load-deformation curves. The moment-curvature diagram for small imperfection amplitude matches the curve which corresponds to the C2 criterion (figure 8.6 of Eurocode [52]) and this criterion could be used. For high imperfection amplitudes, the moment-curvature diagram matches the curve which corresponds to the C3 criterion (figure 8.6 of Eurocode [52]) and thus this should be used to evaluate the bending capacity.

3. Fig. 16c shows the variation of collapse curvature with the amplitude of imperfections. Collapse curvature stabilizes at an imperfection amplitude \( w_0 > 0.60\) much earlier than the collapse moment, which stabilizes at \( w_0 > 1.30\). The important feature of the Fig. 16c is the sudden jump at \( w_0 = 1.30\). This can be explained by the fact that for small imperfection amplitude (\( w_0 < 1.30\)), cylinders do not lose their bending stiffness significantly due to yielding before buckling. As a result, the prebuckling rotation is small. For large imperfection amplitudes (\( w_0 > 1.30\)), the prebuckling loss of bending stiffness is significant due to yielding and consequently, the rotation is large before buckling. As mentioned in the previous section, for high imperfection amplitudes the failure is smooth, while for low imperfection amplitude the failure is abrupt. Based on this observation, the imperfection amplitude can be classified into these two ranges. The critical imperfection amplitude, when the behavior of cylinders changes, can be obtained from Fig. 16c.

To further study the impact of \( R/t \), the load reduction factor \( \alpha \) and collapse curvature reduction factor \( \phi \) are plotted with the amplitude for four \( R/t \) ratios, i.e., 60, 80, 100 and 120 in Fig. 17. The independence of \( \alpha \) with \( R/t \) comes from the fact that the size and the amplitude of imperfection have been scaled to make it independent [58]. In Fig. 17b, the same jumps appear as described previously. Another significant observation is that for higher \( R/t \), the jumps in \( \phi \) occur at the smaller amplitudes.

4.3. Impact of Dimplelike imperfection’s size

The length and the width of the Dimplelike imperfection are taken as \( 2\lambda \) in the previous sections, where \( \lambda \) is classical axisymmetric buckle half-wavelength for cylindrical shells under axial compression [53]. This is done in anticipation that when the dimensions of imperfections are synchronized with eigenmode, they lead to the maximum reduction in load carrying capacity. To verify this assumption, the length and width of the Dimplelike imperfection are parametrically changed for the case of \( R/t = 60 \) and Ramberg-Osgood strain-hardening model \( n = 9 \). First, the length of Dimplelike imperfection is varied (dimension along the axial direction) from \( 2\lambda \) to \( 10\lambda \) with an interval of \( \lambda \) while the other parameters are kept constant, i.e., the width is \( 2\lambda \) and amplitude is \( t/10 \). The results of this analysis are shown in Fig. 18. Fig. 18a shows the moment capacity and Fig. 18b shows the collapse curvature with varying length of the imperfection. It is surprising to see the indifference of moment capacity to the length of the Dimplelike imperfection, while the collapse curvature is increasing with the increase of the length. Fig. 18c and Fig. 18d show the impact of imperfection’s width on the moment capacity and collapse curvature. The moment...
capacity and collapse curvature both are not affected by the width of the Dimplelike imperfection. We found similar results for other \( R_t \) ratios and strain-hardening models used in these studies, i.e., \( R_t = 80, 100, 120 \) and \( n = 13, 30 \). In summary, the moment capacity of thin steel cylindrical shells, which are used in this study, depends primarily on the amplitude of the imperfection and is less dependent on the length and width of the Dimplelike imperfection.

4.4. Effect of strain-hardening models

Fig. 19 shows the impact of the different strain-hardening models on the bending behavior for \( R_t = 60 \). It can be seen that the strain-hardening models affect significantly the peak moment and the collapse curvature. The cylinder has the least capacity when the bilinear model is used and has the maximum capacity for Ramberg-Osgood with \( n = 9 \). The ratios of the peak moment and the collapse curvature for \( n = 9 \) and bilinear are 1.23 and 3.00 respectively which demonstrates the significant role played by strain-hardening models. The collapse curvature is reduced by one third by just changing the hardening model from \( n = 9 \) to bilinear. This is a very crucial finding as sometimes the best fit of a Ramberg-Osgood model is used from the stress-strain tests. This finding suggests that utmost caution should be taken in choosing the strain-hardening model, as it could heavily overestimate or underestimate the bending capacity of thin steel cylindrical shells. The impact of strain-hardening models is more pronounced on the collapse curvature than the peak moment. The highly nonlinear behavior, due to the coupling of material nonlinearity and ovalization, in the vicinity of buckling is responsible for such a drastic impact of strain-hardening models on the collapse curvature.

The impact of strain-hardening models is not the same for all the \( R_t \) ratios. Fig. 20 shows the moment capacities and collapses curvature for four \( R_t \) (60, 80, 100, 120). From Fig. 20, it can be observed that the impact of strain-hardening models is diminishing as \( R_t \) is increasing. This is happening because higher \( R_t \) reduces the collapse curvature and the peak moment. The nonlinear zone in the vicinity of buckling reduces and consequently the role played by strain-hardening models is also reduced. For much higher values of \( R_t \), thin steel cylinders fail elastically, and strain-hardening models do not play any role in deciding the bending capacity of perfect thin cylinders. However, strain-hardening models affect their post-buckling behavior when large deformation appears. Strain-hardening models may have a role in deciding the capacity of imperfect cylinders with the higher \( R_t \) ratios especially if residual stresses are present. But for our range of interest \( (60 < R_t < 120) \), strain-hardening models do have an important role. Fig. 21 shows the moment capacities and collapse curvatures for biased and unbiased imperfect cylinders, and Fig. 22 shows the moment curvatures.
capacities and collapse curvatures for Modal shape and Dimplelike imperfect cylinders. From these figures, it can be concluded that the impact of strain-hardening models on the imperfect cylinders and the perfect cylinder is similar.

5. Validation of finite element modeling

To compare our computational results with the past experiments, we collected the experimental data of Guo et al. [4], Sherman [61], Kiymaz [49], Elchalakani et al. [62] and van Es et al. [39]. The dimensions of cylinders, which are used in these experiments, are different from the dimensions of the cylinders used in this study. For this reason, we compare the non-dimensional load parameter $M_u/M_p$ against the generalized section slenderness $\sqrt{\pi^2} \times D_t f_y$ as used by Guo et al. [4] and Elchalakani et al. [62]. Here $M_u$ is the bending capacity, $M_p$ is the plastic capacity of the cylinder and $f_y$ is the yield stress in Mpa. Fig. 23 shows the plot of $M_u/M_p$ against the generalized section slenderness for the experiments and our computational results. For our computational results, we use Ramberg-Osgood strain-hardening model with $n = 9$ and one perfect and four imperfect cylinders: biased, unbiased, Dimplelike and Modal all with imperfection amplitude $t/10$. The generalized section slenderness varies from 170.40 to 340.80 for our study and the experiments of Guo et al. [4], van Es et al. [39] and Elchalakani et al. [62] have some samples in this range as shown in Fig. 23. A general trend is followed by our computational study, i.e., a reduction of $M_u/M_p$ ratio with the increase of generalized section slenderness. This trend is also observed by the experimental results as can be seen in Fig. 23.

The recent experiments done by Jay et al. [42] have quite large generalized section slenderness and thus to compare our results with these experiments, we have chosen a different non-dimensional parameter, i.e., load reduction factor $\alpha$. The $R/t$ values of this experimental sample vary from 137.50 to 160.50, and so we have created

![Figure 20](image_url)  
**Fig. 20.** Variation of (a) the moment capacity and (b) the collapse curvature with the slenderness $(R/t)$.

![Figure 21](image_url)  
**Fig. 21.** Impact of material-hardening models on the load reduction factor $\alpha$ (a and c) and on the collapse curvature reduction factor $\phi$ (b and d) for biased and unbiased imperfection.
computational cylindrical models having $R/t$ ratios equal to these experiments. Mahmoud et al. [3] reported that imperfection amplitude $t_{0.60}$ shows strong agreement with the experiment. For this reason, load reduction factors $\alpha$ of the computational models are found assuming imperfection amplitude $t_{0.60}$. The results in Fig. 24 show good agreement between the experimental values of $\alpha$ to the computational values of $\alpha$. It should be noted that $\alpha$ for the experiment is calculated as the ratio of the experimental capacity of the specimen by the capacity predicted by a computational model of the specimen without accounting imperfections (perfect geometry). For Dimplelike imperfection, the agreement is quite good, while for biased and unbiased imperfection our $\alpha$ is lower than the experimental value. This discrepancy shows that Dimplelike imperfections are actually the most realistic of all the imperfections accounted in our study. This is evidence to support the recent work undertaken by NASA and others for the update of knockdown factors [63,64].

6. Conclusions

The objective of this study is two-fold: first, to investigate the impact of imperfections and secondly, to observe the influence of strain-
hardening models on thin steel cylindrical shells when the slenderness \( R/t \) varies from 60 to 120. It is found that steel cylindrical shells (60 < \( R/t \) < 120) are highly imperfection sensitive under inelastic bending and that strain-hardening models play a very impactful role on the bending behavior.

Four kinds of geometric imperfections, Modal shape, Dimplelike, unbiased and biased, are applied to make the imperfect cylinders. The presence of imperfections modified many features of the moment-curvature diagram, and reduces the peak moment and the collapse curvature significantly. The reduction in collapse curvature is more pronounced than the reduction in peak moment due to the existence of a highly nonlinear region before buckling. The maximum reduction in load carrying capacity for \( R/t = 60 \), happens in the case of biased imperfection, i.e. around 18%. The reduction in collapse curvature (51%) is also maximum for biased imperfection for \( R/t = 60 \). The influence of Dimplelike imperfection on the bending behavior is minimum among the four imperfections considered in this study. In this case, the reduction in load carrying capacity is around 6% and reduction in collapse curvature is around 30% for \( R/t = 60 \). Similar patterns are observed for other \( R/t \) (80, 100, 120). It can be said that the biased imperfection is worst and the Dimplelike imperfection is least worst among all the imperfections. The location of first yielding in the moment-curvature space and the pre-buckling path are also affected by the geometric imperfections. The first yielding occurs at lower moments for imperfect cylinders as compared to the perfect cylinders. For the biased and the unbiased imperfect cylinders, the pre-buckling paths are different than the pre-buckling path of perfect cylinder whereas the pre-buckling path of the Dimplelike imperfect cylinder and Modal shape imperfect cylinder are same as the perfect cylinder.

The bending behavior of thin steel cylindrical shells highly depends on the strain-hardening models. To investigate this aspect, four strain-hardening models are utilized: a bilinear one and three versions of the Ramberg-Osgood plasticity model. The moment capacity and the collapse curvature is minimum for the bilinear model and maximum for Ramberg-Osgood model with \( n = 9 \) among all the models used. The significance of the strain-hardening model can be understood by the fact that the ratio of collapse curvature for \( n = 9 \) to bilinear is around 3 (\( R/t = 60 \)), which demonstrates that the same cylinder could collapse at three times less curvature if the strain-hardening model changes. This is a very significant finding and it suggests that the selection of a strain-hardening model is extremely important because it can highly overestimate or underestimate the collapse curvature and bending capacity.

This study is limited in many respects as many aspects have not been considered in this study, e.g., the effect of residual stresses, combined loading, discontinuity in cylindrical shell etc. Nonetheless, the results of this study open a new avenue to explore, which will help designers of cylindrical shells and tall wind turbine towers in particular.

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